

1

(a)

$$y = \frac{3x+2}{x-1}$$

$$yx - y = 3x + 2$$

$$(y-3)x = 2+y$$

$$x = \frac{y+2}{y-3}$$

$$f^{-1}(x) = \frac{x+2}{x-3}$$

$$D_{f^{-1}} = R_f = (3, \infty)$$

OR:

$$y = \frac{3x+2}{x-1} = 3 + \frac{5}{x-1}$$

$$y-3 = \frac{5}{x-1}$$

$$x-1 = \frac{5}{y-3}$$

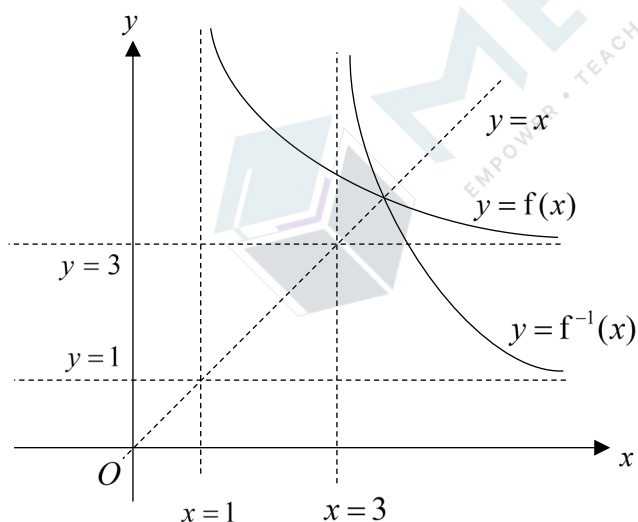
$$x = 1 + \frac{5}{y-3}$$

$$f^{-1}(x) = 1 + \frac{5}{x-3}, \quad D_{f^{-1}} = R_f = (3, \infty)$$

(b)

$$D_f = (1, \infty) \quad D_{f^{-1}} = (3, \infty)$$

$$R_f = (3, \infty) \quad R_{f^{-1}} = (1, \infty)$$



From graph, $f(x) = f^{-1}(x)$ intersects at $y = x$

$$f(x) = x$$

$$\frac{3x+2}{x-1} = x$$

$$x^2 - 4x - 2 = 0$$

$$x = \frac{4 \pm \sqrt{24}}{2}$$

$$= 2 \pm \sqrt{6}$$

$$\text{Since } x > 3, \quad x = 2 + \sqrt{6}$$

OR:

$$f(x) = f^{-1}(x)$$

$$\frac{3x+2}{x-1} = \frac{x+2}{x-3}$$

$$(3x+2)(x-3) = (x+2)(x-1)$$

$$2x^2 - 8x - 4 = 0$$

$$x^2 - 4x - 2 = 0$$

2

(a) Let $y = \frac{3x+18}{2x-3}$.

$$y(2x-3) = 3x+18$$

$$2xy - 3y = 3x + 18$$

$$2xy - 3x = 3y + 18$$

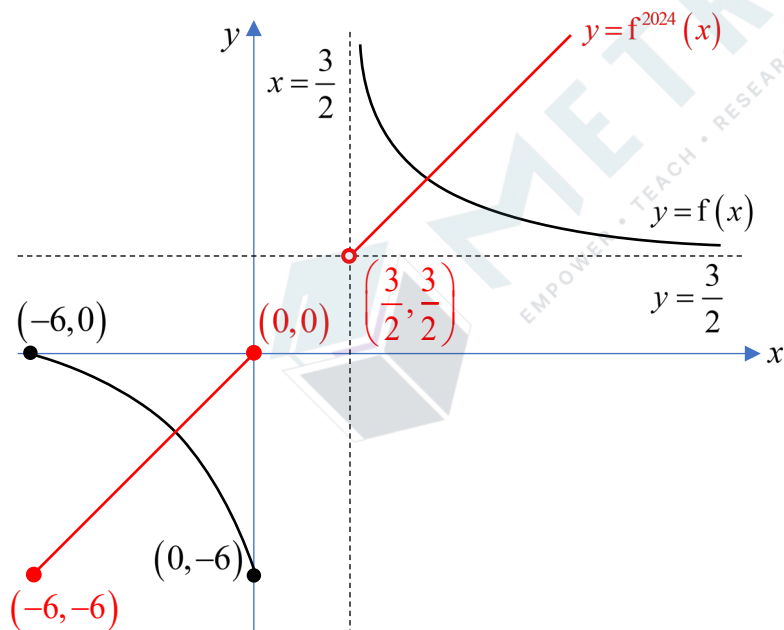
$$x = \frac{3y+18}{2y-3}$$

$$f^{-1}(x) = \frac{3x+18}{2x-3}$$

$$D_{f^{-1}} = R_f = [-6, 0] \cup \left(\frac{3}{2}, \infty\right)$$

(b) $[-6, 0] \cup \left(\frac{3}{2}, \infty\right)$ OR $\left\{x \in \mathbb{R} : -6 \leq x \leq 0, x > \frac{3}{2}\right\}$

(c)



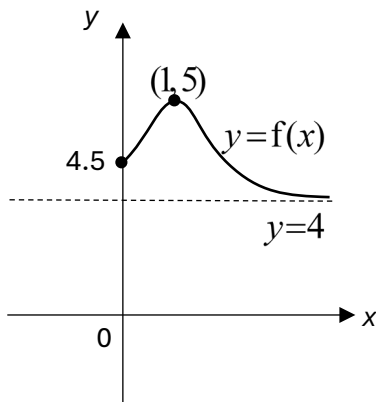
(d) $g^{23}(x) = e^{23}x$

$$g^{23}f(x) = e^{23}f(x)$$

$$R_{g^{23}f} = [-6e^{23}, 0] \cup \left(\frac{3}{2}e^{23}, \infty\right)$$

3

(a)



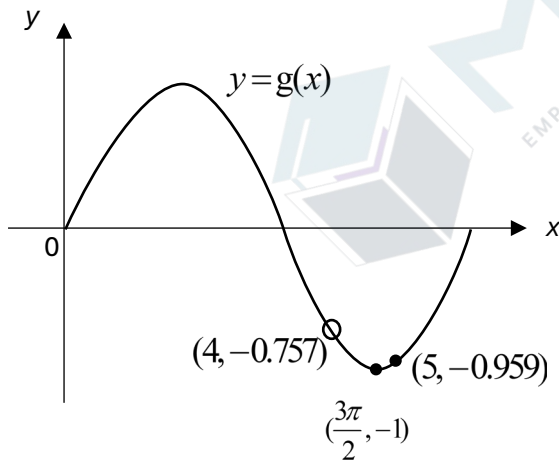
(b) Any 2 points with the same y-value, e.g. any 2 of

$(0, 0)$, $(\pi, 0)$, $(2\pi, 0)$

(c) From graph in part (a), $R_f = (4, 5]$.

Since $R_f = (4, 5] \subseteq [0, 2\pi] = D_g$, the composite function gf exists.

(d)

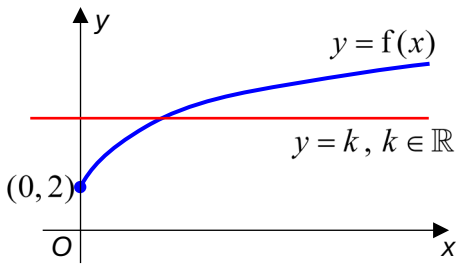


Since $R_f = (4, 5]$, we consider the graph of $y = g(x)$ or $y = \sin x$ restricted to $4 < x \leq 5$.

When $4 < x \leq 5$, $-1 \leq g(x) < -0.757$, so the range of gf is $[-1, -0.757)$.

4

(a)



Let $y = f(x)$. From graph, since any horizontal line $y = k$, $k \in \mathbb{R}$ cuts $y = f(x)$ at most once, $\therefore f$ is 1-1 and hence f^{-1} exists. (shown)

(b) $y = \ln(x+1)^2 + 2$

$$(x+1)^2 = e^{y-2}$$

$$\therefore x = \pm \sqrt{e^{y-2}} - 1$$

$$= e^{\frac{y-2}{2}} - 1 \quad (\text{reject } x = -\sqrt{e^{y-2}} - 1 \text{ since } x \geq 0)$$

$$\therefore f^{-1}(x) = e^{\frac{x-2}{2}} - 1$$

$$D_{f^{-1}} = R_f = [2, \infty)$$

(c) $R_{f^{-1}} = D_f = [0, \infty)$

$$D_{g^{-1}} = R_g = \mathbb{R} \setminus \{-1\}$$

Since $R_{f^{-1}} \subset D_{g^{-1}}$,

$\therefore g^{-1}f^{-1}$ exists.

$$\text{Let } y = \frac{1-x}{x+1}$$

$$xy + y = 1 - x$$

$$x(y+1) = 1 - y$$

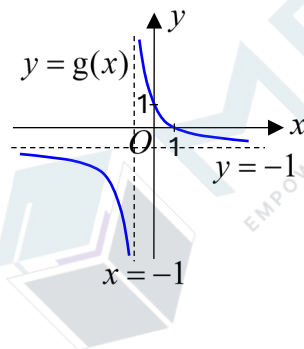
$$\therefore x = \frac{1-y}{y+1}$$

$$\therefore g^{-1}(x) = \frac{1-x}{x+1}$$

Using $R_{f^{-1}} = [0, \infty)$ as restricted domain of g^{-1} ,

$$R_{g^{-1}} = (-1, 1]$$

Hence $R_{g^{-1}f^{-1}} = (-1, 1]$



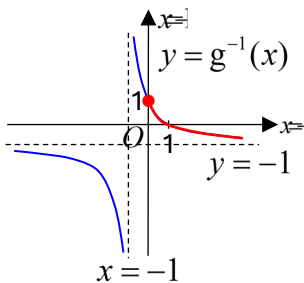
(d) $g^{-1}f^{-1}(x) = \frac{1}{2}$

$$gg^{-1}f^{-1}(x) = g\left(\frac{1}{2}\right)$$

$$\therefore f^{-1}(x) = \frac{1 - \frac{1}{2}}{\frac{1}{2} + 1} = \frac{1}{3}$$

$$ff^{-1}(x) = f\left(\frac{1}{3}\right)$$

$$\Rightarrow x = \ln\left(\frac{1}{3} + 1\right)^2 + 2 = \ln\frac{16}{9} + 2$$



5

(i) For $-4 < x < -1$, $f(x) = -(x+1)^2$

$$\text{Let } y = -(x+1)^2.$$

$$(x+1)^2 = -y$$

$$x+1 = \pm\sqrt{-y}$$

$$x = -1 \pm \sqrt{-y}$$

$$\text{Since } -4 < x < -1, x = -1 - \sqrt{-y}$$

$$\text{For } -1 \leq x \leq 2, f(x) = (x+1)^2$$

$$\text{Let } y = (x+1)^2.$$

$$(x+1)^2 = y$$

$$x+1 = \pm\sqrt{y}$$

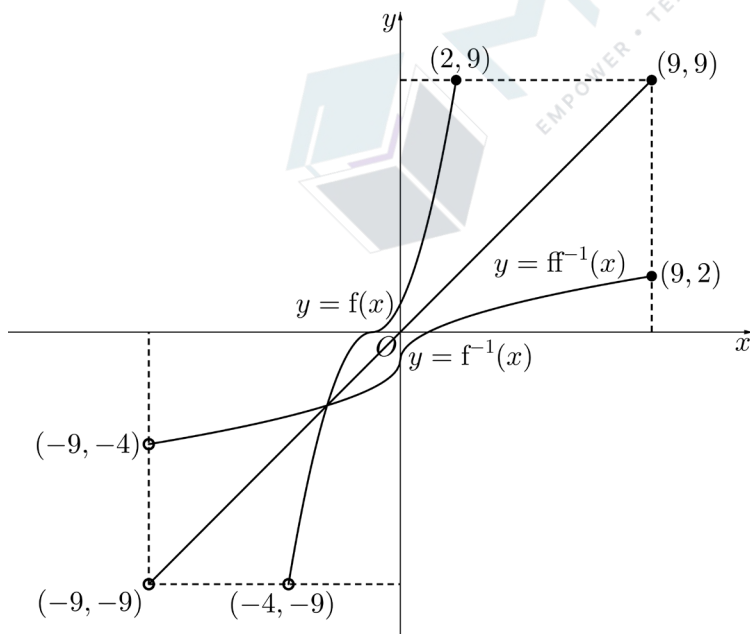
$$x = -1 \pm \sqrt{y}$$

$$\text{Since } -1 \leq x \leq 2, x = -1 + \sqrt{y}$$

Thus,

$$f^{-1}(x) = \begin{cases} -1 - \sqrt{-x}, & \text{for } x \in \mathbb{R}, -9 < x < 0, \\ -1 + \sqrt{x}, & \text{for } x \in \mathbb{R}, 0 \leq x \leq 9. \end{cases}$$

(ii)



(iii) For $f(x) = f^{-1}(x)$, $f(x) = x$.

From the graph, intersection occurs for $-4 < x \leq -1$. Thus

$$-(x+1)^2 = x$$

$$x^2 + 2x + 1 = -x$$

$$x^2 + 3x + 1 = 0$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4}}{2}$$

$$x = \frac{-3 \pm \sqrt{5}}{2}$$

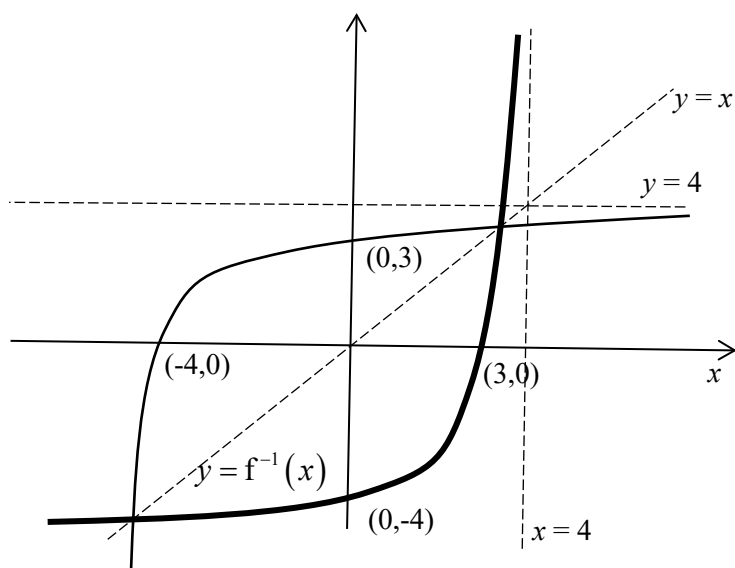
$$x = \frac{-3 - \sqrt{5}}{2} \quad (\because x < -1)$$

Thus, for $f(x) \leq f^{-1}(x)$, $-4 < x \leq \frac{-3 - \sqrt{5}}{2}$.



6

(i)



The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are the reflections of each other about the line $y = x$.

(ii) $R_g = [-4, \infty)$

From graph, $R_{fg} = [0, 4)$

(iii) $g(a) = f^{-1}(0) = -4$

(iv) Let $y = x^2 + 6x + 5$

$$y = x^2 + 6x + 5$$

$$y = (x+3)^2 - 4$$

$$x+3 = \pm\sqrt{y+4}$$

$$x = -3 - \sqrt{y+4} \quad (\text{rej } +\sqrt{y+4} \text{ as } x < -4)$$

$$h^{-1}(x) = -3 - \sqrt{x+4}$$

$$D_{h^{-1}} = (-3, \infty)$$

7

(i) $f(x) = -\frac{x}{2x+1}$ where $D_f = \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

Let $y = f(x)$

$$y = -\frac{x}{2x+1}$$

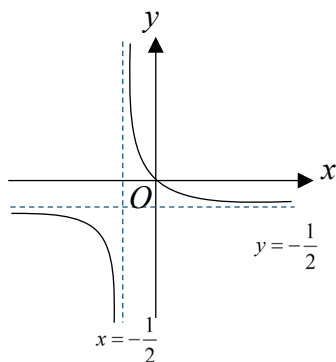
$$2xy + y = -x$$

$$x(2y+1) = -y$$

$$x = -\frac{y}{2y+1}$$

$$f^{-1}(y) = -\frac{y}{2y+1}$$

$$\Rightarrow f^{-1}(x) = -\frac{x}{2x+1}$$



Domain of f^{-1} = Range of $f = \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

Or Domain of $f^{-1} = \mathbb{R} \setminus \left\{-\frac{1}{2}\right\}$

(ii) **Method 1 - Hence**

Since $f^{-1}(x) = -\frac{x}{2x+1} = f(x)$ and the domains $D_f = D_{f^{-1}}$ are the same,

$$f^2(x) = f(f^{-1}(x)) = x$$

Method 2 - Otherwise

$$f^2(x) = f\left(-\frac{x}{2x+1}\right)$$

$$= -\left[\frac{\frac{-x}{2x+1}}{2\left(\frac{-x}{2x+1}\right)+1}\right]$$

$$= \frac{x}{2x+1} \times \frac{2x+1}{-2x+(2x+1)}$$

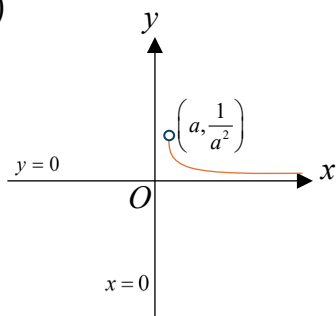
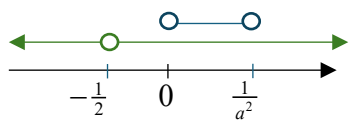
$$= x$$

(iii) $f^{2023}(x) = f(f^{2022}(x))$
 $= f(x)$

$$f^{2023}(5) = f(5) = -\frac{5}{11}$$

(iv) $g(x) = \frac{1}{ax}$ where $D_g = (a, \infty)$

Range of $g = \left(0, \frac{1}{a^2}\right)$



Since $R_g \subseteq D_f$, composite function fg exists.



8

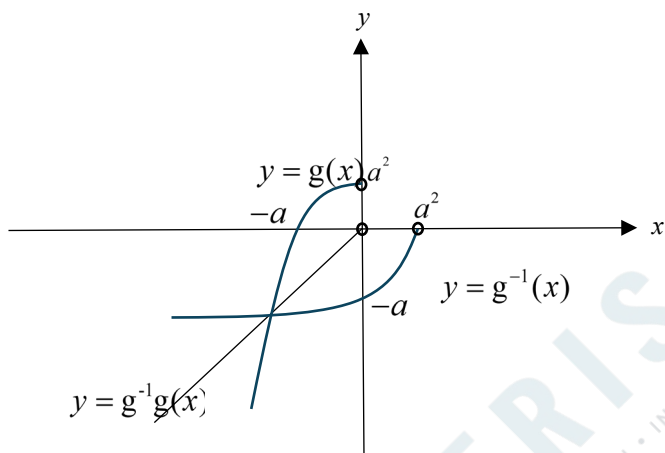
(a) Let $y = -(e^{x-3} - 1) = 1 - e^{x-3} \quad \because x \leq 3$

$$e^{x-3} = 1 - y$$

$$x = 3 + \ln(1 - y)$$

$$f^{-1} : x \mapsto 3 + \ln(1 - x), \quad 0 \leq x < 1$$

(b)



(c) $g(x) = x$

$$a^2 - x^2 = x$$

$$x^2 + x - a^2 = 0$$

$$\left(x + \frac{1}{2}\right)^2 - a^2 - \frac{1}{4} = 0$$

$$x = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + a^2}$$

$$x = -\frac{1}{2} + \sqrt{\frac{1}{4} + a^2} \quad \text{or} \quad x = -\frac{1}{2} - \sqrt{\frac{1}{4} + a^2}$$

(rejected $\because x < 0$)

(d) $R_g = (-\infty, a^2)$

$$D_f = (-\infty, 3]$$

Since $R_g \subseteq D_f$ for $0 < a < 1$, therefore fg exist.

$$R_g = (-\infty, a^2) \xrightarrow{f} R_{fg} = (1 - e^{a^2-3}, 1)$$

9

(a)

$$f(x) = a + \frac{3a}{(x+3)(x-1)}$$

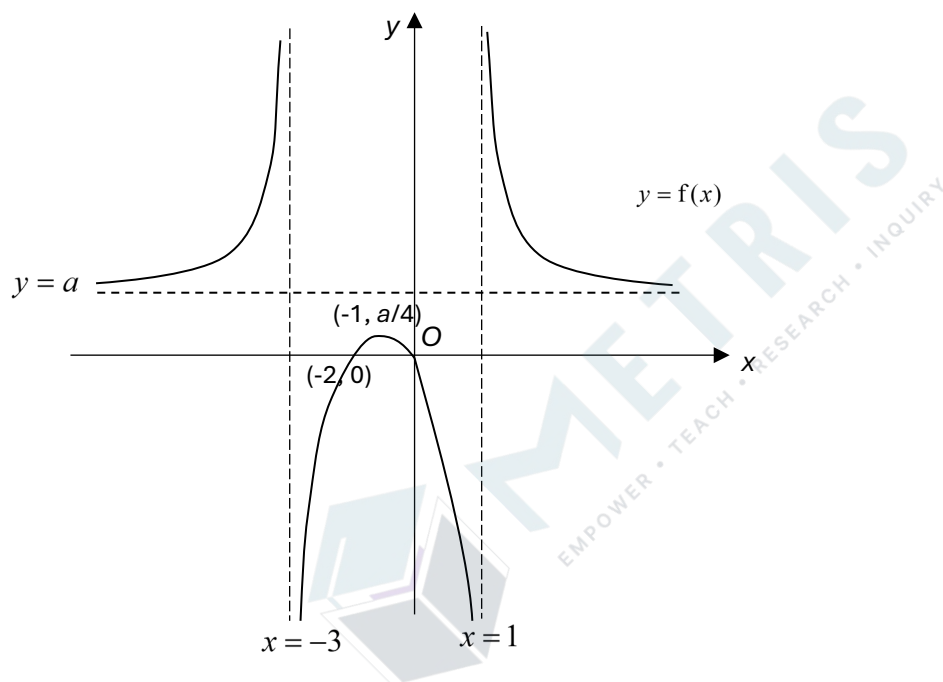
$$f'(x) = \frac{-3a[1(x-1) + 1(x+3)]}{[(x+3)(x-1)]^2} = \frac{-3a[2x+2]}{[(x+3)(x-1)]^2}$$

At stationary point,

$$f'(x) = 0$$

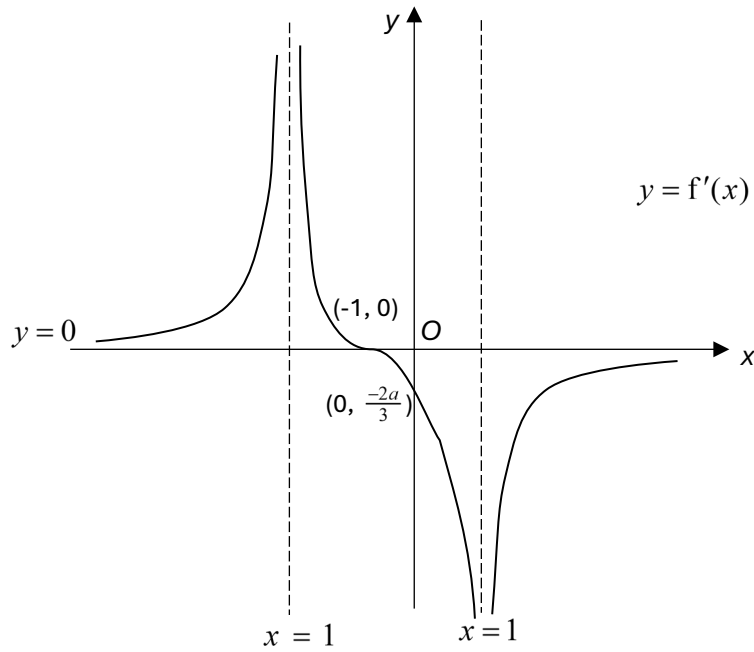
$$\frac{-6a[x+1]}{[(x+3)(x-1)]^2} = 0 \Rightarrow x = -1$$

(b)



(c) $-3 < x < -1$

(d)



(e) $D_g = [2, \infty)$

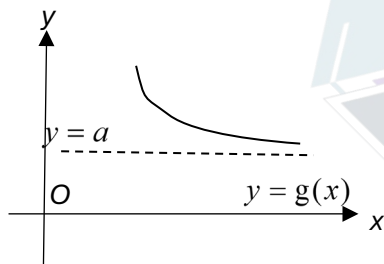
$$x = 2, y = a + \frac{3a}{(2+3)(2-1)} = \frac{8}{5}a$$

$$R_g = (a, \frac{8}{5}a]$$

For g to exist, $R_g \subseteq D_g$.

Hence, the condition on a is $a \geq 2$.

(f)



$$[2, \infty) \rightarrow (a, \frac{8}{5}a] \rightarrow [g(\frac{8}{5}a), g(a))$$

For $a = 5$,

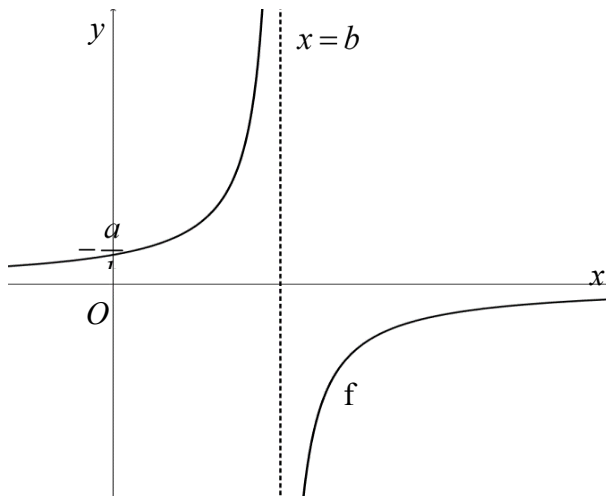
$$g(\frac{8}{5}a) = g(8) = 5 + \frac{3(5)}{(8+3)(8-1)} = 5\frac{15}{77} \quad (= \frac{400}{77})$$

$$g(a) = g(5) = 5 + \frac{3(5)}{(5+3)(5-1)} = 5\frac{15}{32} \quad (= \frac{175}{32})$$

$$\therefore R_{gg} = [5\frac{15}{77}, 5\frac{15}{32})$$

10

(a)



$D_f = \mathbb{R} \setminus \{b\}$ and $R_f = \mathbb{R} \setminus \{0\}$. Since $R_f \not\subseteq D_f$, ff does not exist.

(b)(i)

The range of g contains all the real numbers except 0 and $\frac{-b^2}{0-b} = b$. Thus

$$R_g = \mathbb{R} \setminus \{0, b\} = D_g \subseteq D_g$$

For gg :

$$\begin{aligned} gg(x) &= g\left(-\frac{b^2}{x-b}\right) \\ &= -\frac{b^2}{-\frac{b^2}{x-b}-b} \\ &= \frac{b^2(x-b)}{b^2+b(x-b)} \\ &= \frac{b(x-b)}{b+x-b} \\ &= \frac{bx-b^2}{x} \end{aligned}$$

For g^{-1} :

$$\begin{aligned} \text{Let } y &= -\frac{b^2}{x-b} \\ xy-by &= -b^2 \\ x &= \frac{by-b^2}{y} \\ g^{-1}(y) &= \frac{by-b^2}{y} \\ g^{-1}(x) &= \frac{bx-b^2}{x} \end{aligned}$$

Thus they have the same rule.

(b)(ii)

$$\begin{aligned} gg(x) &= g^{-1}(x) \\ g[gg(x)] &= g[g^{-1}(x)] \\ g^3(x) &= x \\ 2024 &= 674 \times 3 + 2 \\ g^{2024}(x) &= gg(x) \\ g^{2024}(2b) &= gg(2b) \\ &= \frac{b(2b)-b^2}{2b} \\ &= \frac{1}{2}b \end{aligned}$$

$$\text{Let } y = \frac{ax+b}{x-a}.$$

$$xy - ay = ax + b$$

$$x(y-a) = ay + b$$

$$x = \frac{ay+b}{y-a}$$

$$\therefore f^{-1}(x) = \frac{ax+b}{x-a}$$

f is self-inverse.

Symmetrical about $y = x$.

$$R_f = \mathbb{R} \setminus \{a\}$$

$$\text{Since } D_g = \mathbb{R}, R_f \subseteq D_g.$$

$\therefore$ gf exists.

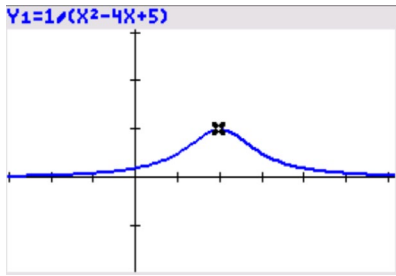
$$D_{gf} = D_f = \mathbb{R} \setminus \{a\}$$

$$\mathbb{R} \setminus \{a\} \xrightarrow{f} \mathbb{R} \setminus \{a\} \xrightarrow{g} (0, c) \cup (c, e) \text{ or } (0, e) \setminus \{c\}$$



12

(a)



$$f(x) = \frac{1}{x^2 - 4ax + 5a^2}$$

$$f'(x) = \frac{2x - 4a}{(x^2 - 4ax + 5a^2)^2}$$

For maximum point,

$$\frac{2x - 4a}{(x^2 - 4ax + 5a^2)^2} = 0$$

$$2x - 4a = 0$$

$$x = 2a$$

Hence **largest** $k = 2a$

Alternative method:

$$x^2 - 4ax + 5a^2$$

$$= (x - 2a)^2 - (2a)^2 + 5a^2$$

$$= (x - 2a)^2 + a^2$$

Since $x = 2a$ gives the minimum value of $x^2 - 4ax + 5a^2$,

it gives the maximum value for $f(x) = \frac{1}{x^2 - 4ax + 5a^2}$.

Hence **largest** $k = 2a$

(b) When $x = a$,

$$f(a) = \frac{1}{a^2 - 4a(a) + 5a^2} = \frac{1}{2a^2}$$

From the graph, $0 < f(x) < \frac{1}{2a^2}$

$$\therefore R_f = \left(0, \frac{1}{2a^2}\right)$$

To show f^2 exists, $R_f \subseteq D_f$.

Since, $a > 1 \Rightarrow \frac{1}{a} < 1 \Rightarrow \frac{1}{2a^2} < \frac{1}{2} \Rightarrow \frac{1}{2a^2} < a$,

$$\therefore R_f = \left(0, \frac{1}{2a^2}\right) \subseteq D_f = (-\infty, a).$$

Thus f^2 exists. (Shown)

(c) Let $y = \frac{1}{x^2 - 4ax + 5a^2}$

$$y = \frac{1}{(x-2a)^2 + a^2}$$

$$(x-2a)^2 + a^2 = \frac{1}{y}$$

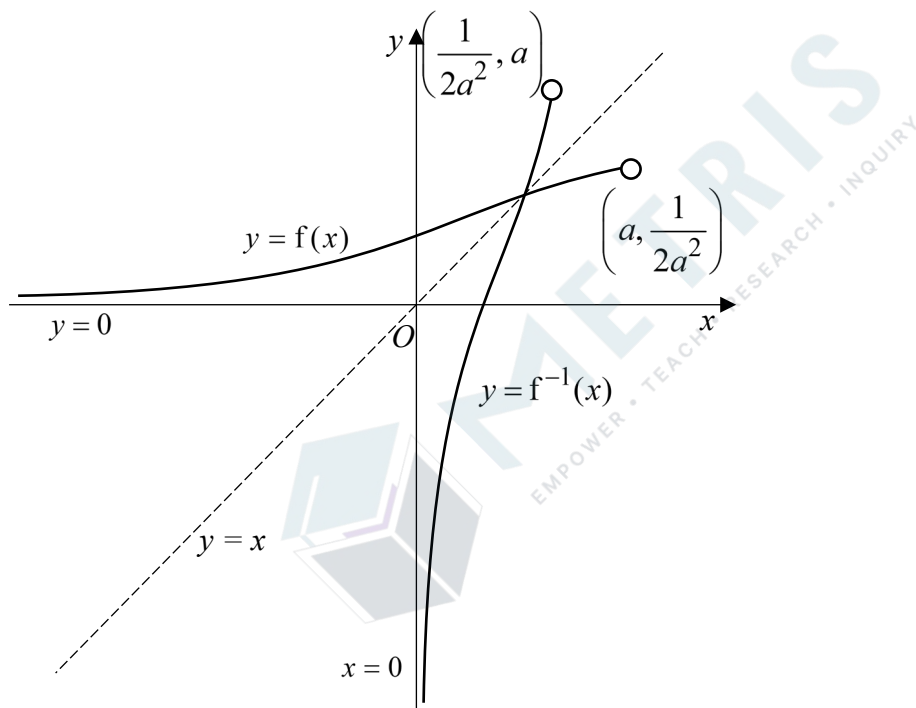
$$x-2a = \pm \sqrt{\frac{1}{y} - a^2}$$

$$x = 2a \pm \sqrt{\frac{1}{y} - a^2}$$

Since $x < a$, hence $x = 2a - \sqrt{\frac{1}{y} - a^2}$

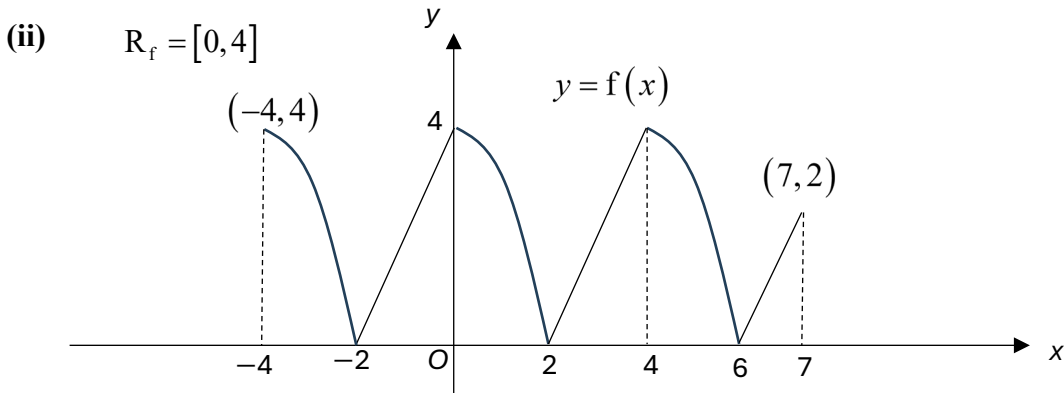
Hence $f^{-1}(x) = 2a - \sqrt{\frac{1}{x} - a^2}$

(d)



13

(i) $f(45) = f(41) = f(37) = \dots = f(1) = 4 - (1)^2 = 3$



(iii) $R_f = [0, 4] \not\subset D_g = (0, 1)$
Hence, gf does not exist.

(iv) $g : x \mapsto (x-1)^2 + 2$ for $x \in \mathbb{R}, 0 < x < 1$.

$R_g = (2, 3)$

$fg(x) = 2[(x-1)^2 + 2] - 4 = 2(x-1)^2$

Let $(fg)^{-1}(\frac{1}{2}) = \alpha$

$fg(\alpha) = \frac{1}{2}$

$2(\alpha-1)^2 = \frac{1}{2}$

$(\alpha-1)^2 = \frac{1}{4}$

$(\alpha-1) = \frac{1}{2}$ or $-\frac{1}{2}$

$\alpha = \frac{3}{2}$ or $\frac{1}{2}$

Since $D_{fg} = D_g = (0, 1)$, $\alpha = \frac{1}{2}$

Alternative Method

$y = 2(x-1)^2, 0 < x < 1$

$(x-1)^2 = \frac{y}{2}$

$(x-1) = \pm \sqrt{\frac{y}{2}}$

$x = 1 \pm \sqrt{\frac{y}{2}}$

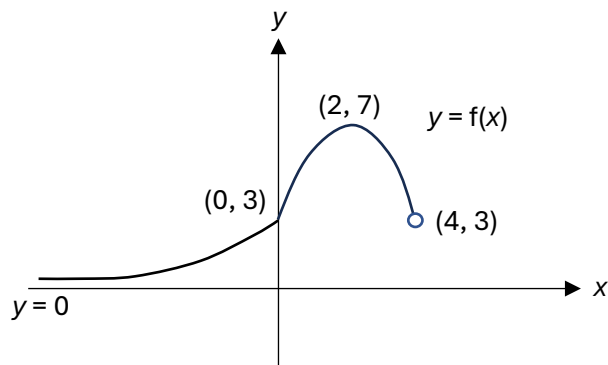
Since $0 < x < 1$, $x = 1 - \sqrt{\frac{y}{2}}$

$(fg)^{-1}(x) = 1 - \sqrt{\frac{x}{2}}$

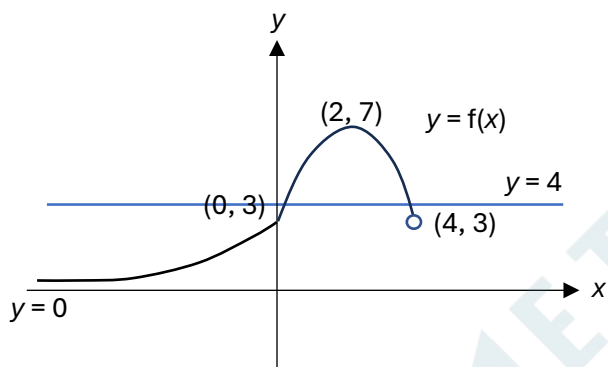
$(fg)^{-1}(\frac{1}{2}) = 1 - \sqrt{\frac{(\frac{1}{2})}{2}} = \frac{1}{2}$

14

(a)



(b) Method 1: Graph



The line $y = 4$ cuts the graph $y = f(x)$ more than once. f is not one-one. Hence, f inverse does not exist.

Method 2: Observation

$$f(1) = 3 + 4(1) - (1)^2 = 6$$

$$f(3) = 3 + 4(3) - (3)^2 = 6$$

Since $f(1) = f(3)$, f is not one-one. f inverse does not exist.

(c) Largest $k = 2$

(d) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(e)

$$y = 3e^x$$

$$e^x = \frac{y}{3}$$

$$x = \ln\left(\frac{y}{3}\right) \quad \text{since } y > 0$$

$$f^{-1}(x) = \ln\left(\frac{x}{3}\right), \quad D_{f^{-1}} = R_f = (0, 3]$$

$$y = 3 + 4x - x^2$$

$$x^2 - 4x + (y - 3) = 0$$

$$\begin{aligned} x &= \frac{4 \pm \sqrt{16 - 4(y - 3)}}{2} \\ &= 2 \pm \sqrt{7 - y} \end{aligned}$$

$$\text{Since } x < 2, \quad x = 2 - \sqrt{7 - y}$$

$$f^{-1}(x) = 2 - \sqrt{7 - x}, \quad D_{f^{-1}} = R_f = (3, 7)$$

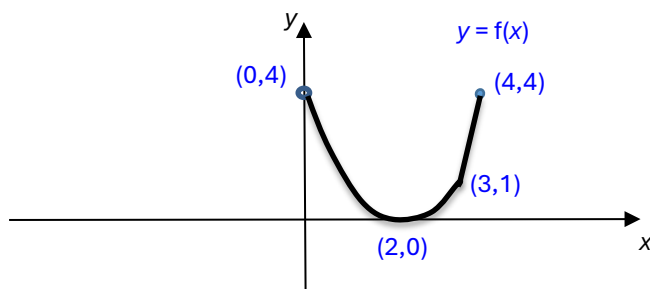
$$f^{-1}(x) = \begin{cases} \ln\left(\frac{x}{3}\right) & \text{for } 0 < x \leq 3, \\ 2 - \sqrt{7 - x} & \text{for } 3 < x < 7. \end{cases}$$



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(a)(i)



The line $y = 2$ cuts the graph of $y = f(x)$ twice.

Hence f is not a one-one function and so f^{-1} does not exist.

(a)(ii) $k = 2$

$$y = (x-2)^2$$

$$x = 2 - \sqrt{y} \quad \text{or} \quad 2 + \sqrt{y} \quad (\text{reject as } 0 < x \leq 2)$$

$$\therefore f^{-1}(x) = 2 - \sqrt{x}, \quad 0 \leq x < 4$$

(a)(iii) For composite fg to exist, $R_g \subseteq D_f$

From graph/observation, $R_g = (3, 4]$.

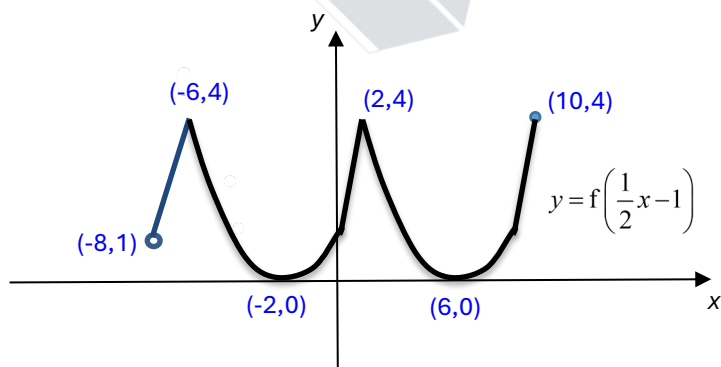
Hence, only second 'piece' of f is relevant: $3x - 8, \quad 3 < x \leq 4$

Therefore, $fg : x \mapsto 3(e^x + 3) - 8, \quad x \in \mathbb{R}, \quad x \leq 0$

(b)(i) $f(25) = f(1 + 6 \times 4) = f(1) = 1$

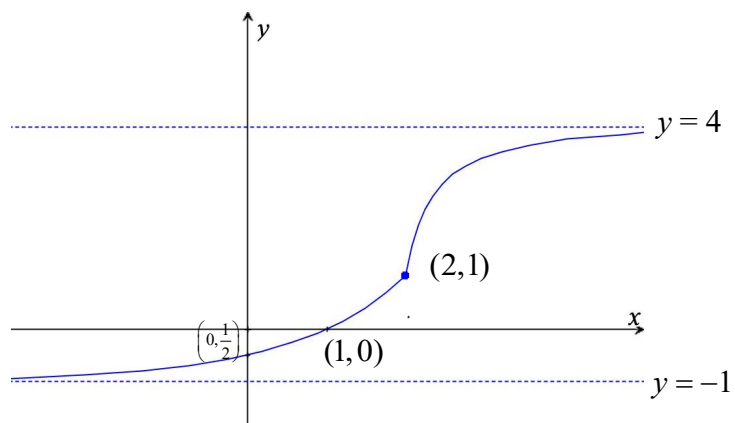
$$f(-8) = f(4 - 3 \times 4) = f(4) = 4$$

(b)(ii)

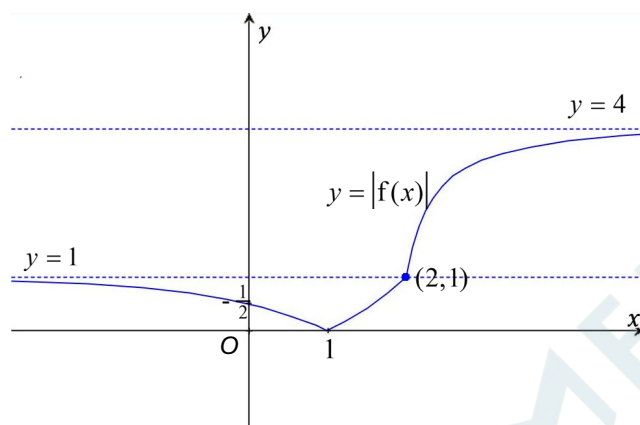


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(i)



(ii)



maximum $a = 1$



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$$(iii) \quad x < 1 \Rightarrow g(x) = -\left[\frac{1}{2}(2^x) - 1\right]$$

$$g(x) = 1 - \frac{1}{2}(2^x)$$

$$\text{Let } y = 1 - \frac{1}{2}(2^x)$$

$$2^x = 2(1 - y)$$

$$x = \log_2(2(1 - y))$$

$$x = 1 + \log_2(1 - y)$$

$$\text{Since } x = g^{-1}(y),$$

$$\therefore g^{-1}(y) = 1 + \log_2(1 - y)$$

$$\therefore g^{-1}(x) = 1 + \log_2(1 - x)$$

$$D_{g^{-1}} = R_g = (0, 1)$$

$$(iv) \quad f(x) = 2$$

$$\Rightarrow 4 - \frac{3}{3x-5} = 2$$

$$\Rightarrow x = \frac{13}{6}$$

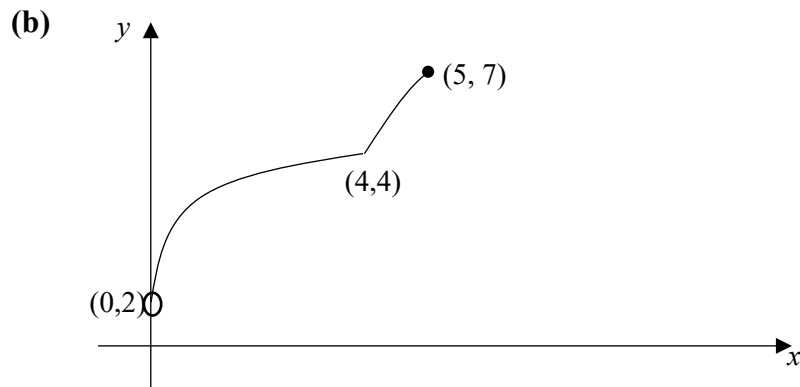
$$f^2(x) = \begin{cases} \frac{1}{2}\left(2^{\frac{1}{2}(2^x)-1}\right) - 1 & \text{for } x \in \mathbb{R}, x < 2, \\ \frac{1}{2}\left(2^{4-\frac{3}{3x-5}}\right) - 1 & \text{for } x \in \mathbb{R}, 2 \leq x < \frac{13}{6}, \\ 4 - \frac{3}{3\left(4-\frac{3}{3x-5}\right)-5} & \text{for } x \in \mathbb{R}, x \geq \frac{13}{6}. \end{cases}$$

$$= \begin{cases} 2^{2^x-2} - 1 & \text{for } x \in \mathbb{R}, x < 2, \\ 2^{3-\frac{3}{3x-5}} - 1 & \text{for } x \in \mathbb{R}, 2 \leq x < \frac{13}{6}, \\ 4 - \frac{9x-15}{21x-44} & \text{for } x \in \mathbb{R}, x \geq \frac{13}{6}. \end{cases}$$

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(a) $2x - x^2 - 28 = 8 - (x-6)^2$

Therefore the largest value of $a = 6$



(c) Let $y = 2 + \sqrt{x}$
 $x = (y-2)^2$

For $4 \leq x \leq 5$

$$y = 8 - (x-6)^2$$

$$(x-6)^2 = 8-y$$

$$x = 6 \pm \sqrt{8-y}$$

Since $x \leq 5$

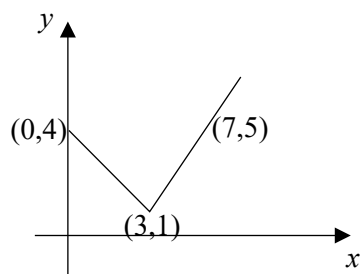
$$x = 6 - \sqrt{8-y}, \quad 4 \leq y \leq 7$$

$$\therefore f^{-1}: x \mapsto \begin{cases} (x-2)^2, & 2 < x < 4 \\ 6 - \sqrt{8-x}, & 4 \leq x \leq 7 \end{cases}$$

(d) Since $R_f = (2, 7] \subset D_g = \mathbb{R}^+$, $\therefore$ gf exists.

$$(0, 5] \xrightarrow{f} (2, 7] \xrightarrow{g} [1, 5]$$

$$R_{gf} = [1, 5]$$



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$$(a) \quad a(-2)^2 - 2b + c = 17 \Rightarrow 4a - 2b + c = 17 \quad \text{---(1)}$$

$$a\left(\frac{1}{2}\right)^2 + \frac{1}{2}b + c = \frac{3}{4} \Rightarrow \frac{1}{4}a + \frac{1}{2}b + c = \frac{3}{4} \quad \text{---(2)}$$

$$a(5)^2 + 5b + c = 3 \Rightarrow 25a + 5b + c = 3 \quad \text{---(3)}$$

Using GC, $a = 1$, $b = -5$, $c = 3$.

$$y = x^2 - 5x + 3$$

$$(b) \quad \text{Let } y = f(x) = x^2 - 5x + 3, \quad x \leq 0$$

$$y = x^2 - 5x + 3$$

$$y = \left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 3$$

$$y + \frac{13}{4} = \left(x - \frac{5}{2}\right)^2$$

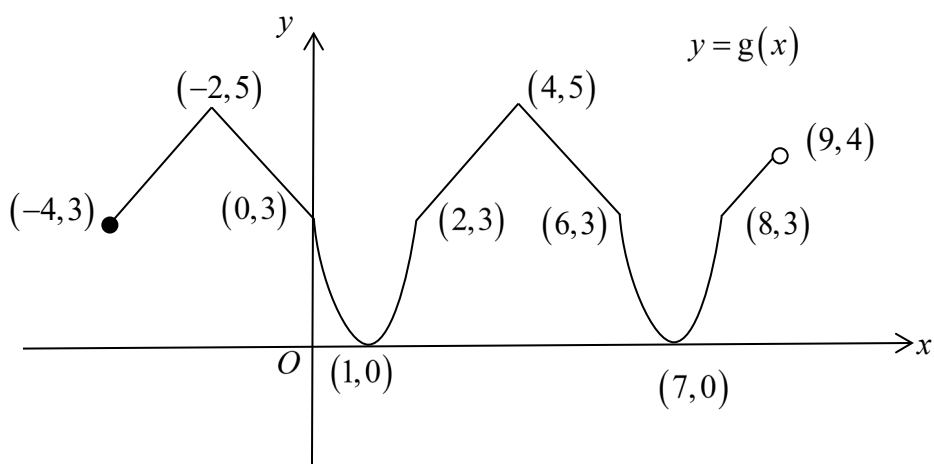
$$x - \frac{5}{2} = \pm \sqrt{y + \frac{13}{4}}$$

$$x = \frac{5}{2} \pm \sqrt{y + \frac{13}{4}}$$

$$x = \frac{5}{2} - \sqrt{y + \frac{13}{4}} \quad (\text{since } x \leq 0)$$

$$f^{-1}(x) = \frac{5}{2} - \sqrt{x + \frac{13}{4}}$$

(c)



(d) $0 \leq x \leq \pi$

$$-\frac{3}{2} \leq \frac{3}{2} \cos x \leq \frac{3}{2}$$

$$0 \leq \frac{3}{2} + \frac{3}{2} \cos x \leq 3$$

Since $R_h = [0, 3] \subseteq [0, 6) = D_g$, therefore the function gh exists.

$$\begin{array}{ccccc} D_h & \xrightarrow{h} & R_h & \xrightarrow{g} & R_{gh} \\ [0, \pi] & & [0, 3] & & [0, 4] \\ & \text{Restricted domain of } g & & & \\ R_{gh} = [0, 4] & & & & \end{array}$$

