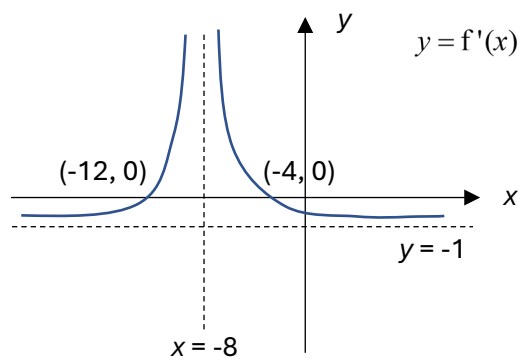
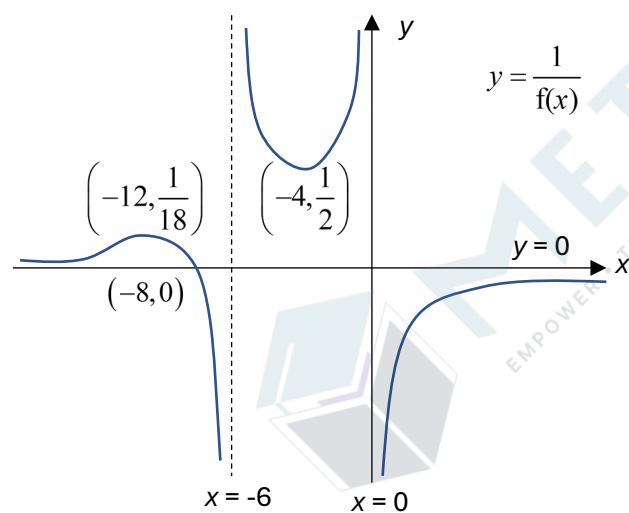


1

(a)



(b)



2

$$\begin{aligned}f(x) &= \ln\left(\frac{a}{x+3a}\right) \\&= \ln a - \ln(x+3a)\end{aligned}$$

$$\begin{aligned}y = \ln x &\xrightarrow{A} y = \ln(x+3a) \\&\xrightarrow{B} y = -\ln(x+3a) \xrightarrow{C} y = \ln a - \ln(x+3a)\end{aligned}$$

Sequence of transformations:

A: A translation of $3a$ units in the negative x -direction

B: A reflection in the x -axis

C: A translation of $\ln a$ units in the positive y -direction

OR

$$\begin{aligned}y = \ln x &\xrightarrow{(1)} y = -\ln x \\&\xrightarrow{(2)} y = -\ln(x+3a) \xrightarrow{(3)} y = \ln a - \ln(x+3a)\end{aligned}$$

Sequence of transformations:

(1): A reflection in the x -axis

(2): A translation of $3a$ units in the negative x -direction

(3): A translation of $\ln a$ units in the positive y -direction



3

(a) Method 1

$$y = e^x \rightarrow y = -e^x \rightarrow y = -e^x + 2$$

Reflect about the x -axis.

Translate by 2 units in the positive y -direction.

Method 2

$$y = e^x \rightarrow y = e^x - 2 \rightarrow y = -(e^x - 2) = -e^x + 2$$

Translate by 2 units in the negative y -direction.

Reflect about the x -axis.

(b) After A : $y = 3x + 6$

$$\text{After } B : y = 3(x - 2) + 6 = 3x$$

$$\text{After } C : y = \frac{1}{3}(3x) = x$$

(c)(i) Asymptotes : $x = a$, $y = \frac{1}{d}$

$$\text{Axial intercepts : } (c, 0), \left(0, \frac{1}{b}\right)$$

(c)(ii) Asymptotes : $x = d$, $y = c$

$$\text{Axial intercepts : } (0, a), (b, 0)$$



(a) Method 1:

$$\begin{aligned}
 y = f(3x - 2) + a & \xrightarrow{\text{Replace } y \text{ with } y+a} y = f(3x - 2) \\
 & \xrightarrow{\text{Replace } x \text{ with } \frac{x}{3}} y = f(x - 2) \\
 & \xrightarrow{\text{Replace } x \text{ with } x+2} y = f(x)
 \end{aligned}$$

1. Translate in negative y -direction by a units.
2. Scale parallel to x -axis with scale factor 3.
3. Translate in negative x -direction by 2 units.

Method 2:

$$\begin{aligned}
 y = f(3x - 2) + a & \xrightarrow{\text{Replace } y \text{ with } y+a} y = f(3x - 2) \\
 & \xrightarrow{\text{Replace } x \text{ with } x+\frac{2}{3}} y = f(3x) \\
 & \xrightarrow{\text{Replace } x \text{ with } \frac{x}{3}} y = f(x)
 \end{aligned}$$

1. Translate in negative y -direction by a units.
2. Translate in negative x -direction by $\frac{2}{3}$ units.
3. Scale parallel to x -axis with scale factor 3.

(b) Method 1:

$$\begin{aligned}
 & (b, 0) \\
 & \downarrow \text{ Translate in negative } y\text{-direction by } a \text{ units} \\
 & (b, -a) \\
 & \downarrow \text{ Scale parallel to } x\text{-axis with scale factor 3} \\
 & (3b, -a) \\
 & \downarrow \text{ Translate in negative } x\text{-direction by 2 units} \\
 & (3b - 2, -a) \\
 & \downarrow \text{ Transform } y = f(x) \text{ to } y = \frac{1}{f(x)} \\
 & (3b - 2, -\frac{1}{a})
 \end{aligned}$$

$$\therefore c = 3b - 2, \quad d = -\frac{1}{a}$$

Method 2:

$$\begin{aligned}
 & (b, 0) \\
 & \downarrow \text{ Translate in negative } y\text{-direction by } a \text{ units} \\
 & (b, -a) \\
 & \downarrow \text{ Translate in negative } x\text{-direction by } \frac{2}{3} \text{ units} \\
 & (b - \frac{2}{3}, -a) \\
 & \downarrow \text{ Scale parallel to } x\text{-axis with scale factor 3} \\
 & (3b - 2, -a) \\
 & \downarrow \text{ Transform } y = f(x) \text{ to } y = \frac{1}{f(x)} \\
 & (3b - 2, -\frac{1}{a})
 \end{aligned}$$

$$\therefore c = 3b - 2, \quad d = -\frac{1}{a}$$

5

(a) $y = f(x) \xrightarrow{\text{scale}} \frac{y}{3} = f(x) \xrightarrow{\text{translate}} \frac{y}{3} = f(x+4)$

So the equation of C is $y = 3f(x+4)$.

Since we know $f(7) = -9$, substituting $x = 3$ we get $y = -27$. Hence the corresponding point is $(3, -27)$.

Additionally, we are given that $f'(7) = 10$. So differentiating the equation for C we get $y' = 3f'(x+4)$. Again, substituting $x = 3$ we get $y' = 30$ i.e. the gradient at this point is 30.

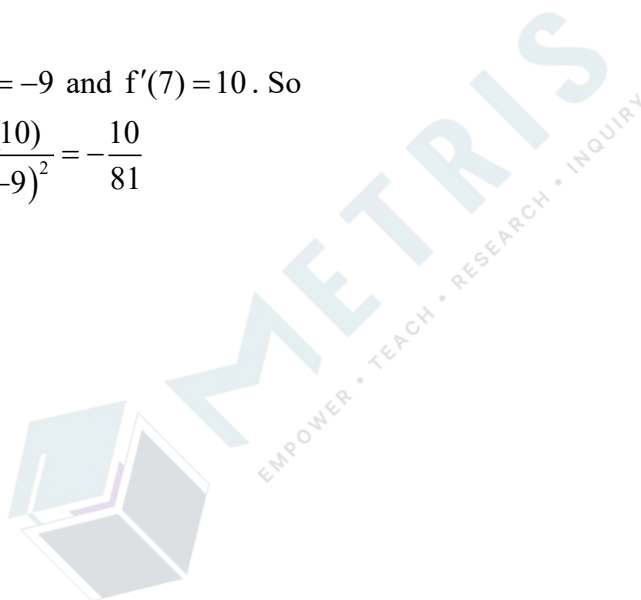
(b) The corresponding point is $(7, -\frac{1}{9})$.

Differentiate $y = \frac{1}{f(x)}$:

$$\frac{dy}{dx} = -\frac{f'(x)}{[f(x)]^2}$$

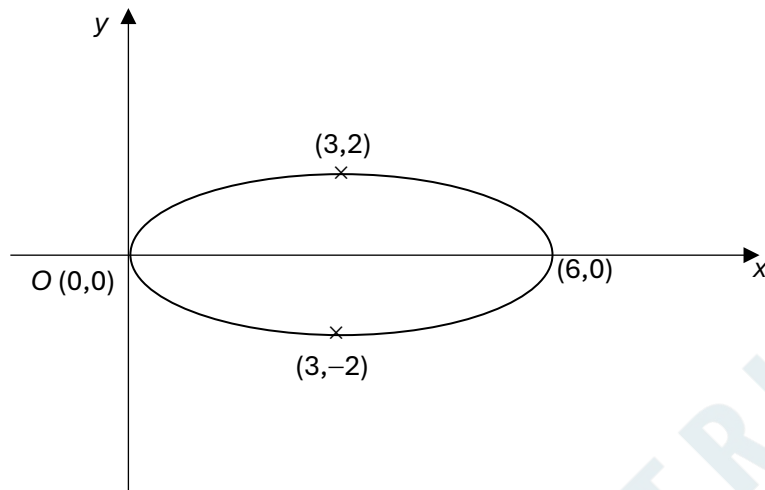
We are given that $f(7) = -9$ and $f'(7) = 10$. So

$$\text{At } x = 7, \left. \frac{dy}{dx} \right|_{x=7} = -\frac{(10)}{(-9)^2} = -\frac{10}{81}$$



6

(a) $4x^2 - 24x + 9y^2 = 0.$
 $4(x^2 - 6x) + 9y^2 = 0$
 $4(x-3)^2 + 9y^2 = 36$
 $\frac{(x-3)^2}{3^2} + \frac{y^2}{2^2} = 1$



(b) The required sequence of transformations is

- A: A translation of magnitude 3 unit in the negative direction of the x -axis.
 B: A translation of magnitude 1 unit in the positive direction of the y -axis.
 C: A scaling parallel to the y -axis by a scale factor of $\frac{1}{\alpha}$.

OR

- A: A translation of magnitude 3 unit in the negative direction of the x -axis.
 B: A **scaling parallel to the y -axis** by a scale factor of $\frac{1}{\alpha}$.
 C: A **translation of magnitude $\frac{1}{\alpha}$** unit in the positive direction of the y -axis.

(c) $\frac{x^2}{9} + \frac{(\alpha y - 1)}{4} = 1$
 $\frac{x^2}{3^2} + \frac{(y - 1/\alpha)^2}{(2/\alpha)^2} = 1$

For shape of a circle, $\alpha = \frac{2}{3}$

7

(a)

$$y = \frac{1}{x}$$

↓ Replace x by $x - a$

$$y = \frac{1}{x - a}$$

↓ Replace y by $\frac{y}{3a}$

$$y = \frac{3a}{x - a}$$

↓ Replace y by $y - 2$

$$y = 2 + \frac{3a}{x - a}$$

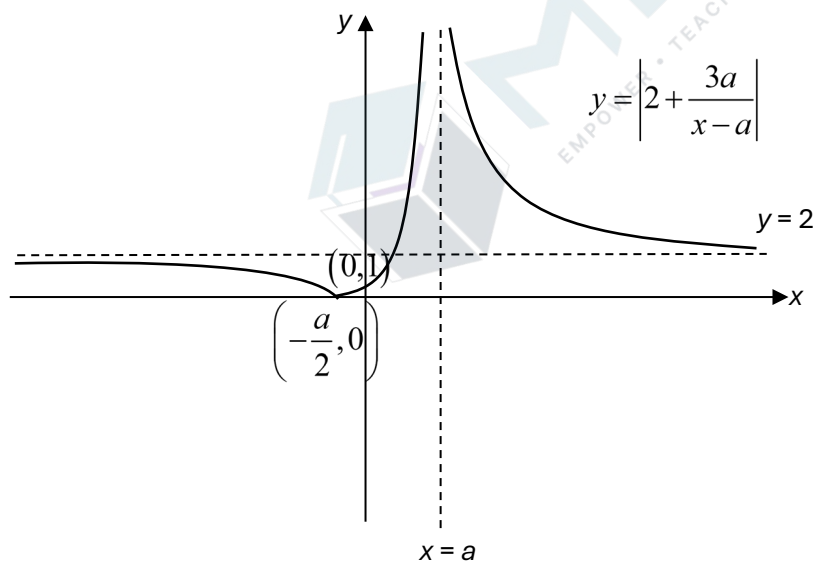
Note $a > 0$.

1. Translation of a units in the positive x -direction.
2. Stretch by factor $3a$ parallel to the y -axis.
3. Translation of 2 units in the positive y -direction.

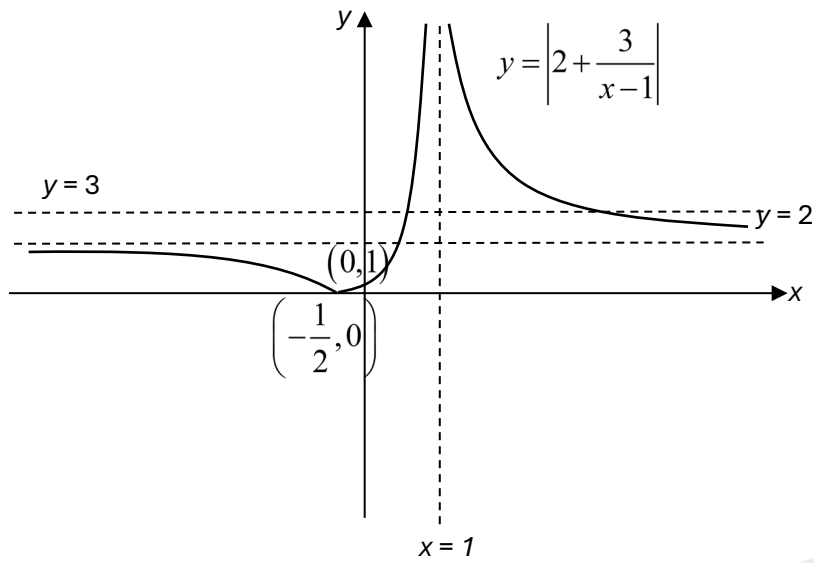
OR

1. Stretch by factor $3a$ parallel to the x -axis.
2. Translation of a units in the positive x -direction.
3. Translation of 2 units in the positive y -direction.

(b)



(c)



$$\left| 2 + \frac{3}{x-1} \right| = 3$$

$$2 + \frac{3}{x-1} = \pm 3$$

$$2 + \frac{3}{x-1} = 3 \quad \text{or} \quad 2 + \frac{3}{x-1} = -3$$

$$\frac{3}{x-1} = 1$$

$$x - 1 = 3$$

$$x = 4$$

$$\frac{3}{x-1} = -5$$

$$-5(x-1) = 3$$

$$x = \frac{2}{5}$$

For $\left| 2 + \frac{3}{x-1} \right| < 3$, from the graph in part (b),

$$x < \frac{2}{5} \quad \text{or} \quad x > 4$$

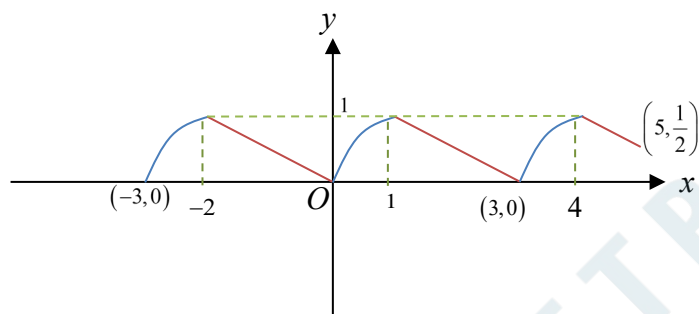
8

(a)(i) Horizontal asymptote: $y = 2b$
 x -intercept: $(a-1, 0)$

(a)(ii) Horizontal asymptote: $y = b$
 x -intercept: $(-a, 0)$

(a)(iii) Horizontal asymptote: $y = \frac{1}{b}$
 x -intercept: inconclusive
(it would result in a vertical asymptote at $x = a$)

(b)(i)



(b)(ii) $\int_{-3}^5 f(x) \, dx$ refers to the area under the curve from $x = -3$ to $x = 5$.

Method 1

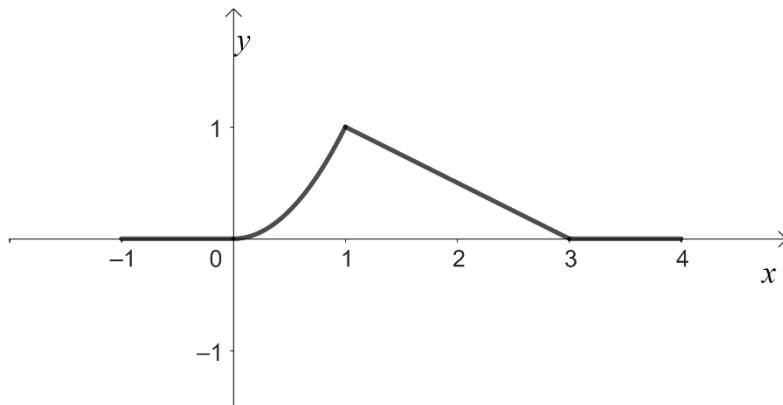
$$\begin{aligned} \int_{-3}^5 f(x) \, dx &= 3 \int_0^1 \sqrt{x} \, dx + 2 \int_1^3 -\frac{1}{2}x + \frac{3}{2} \, dx + \int_1^2 -\frac{1}{2}x + \frac{3}{2} \, dx \\ &= 4.75 \end{aligned}$$

Method 2

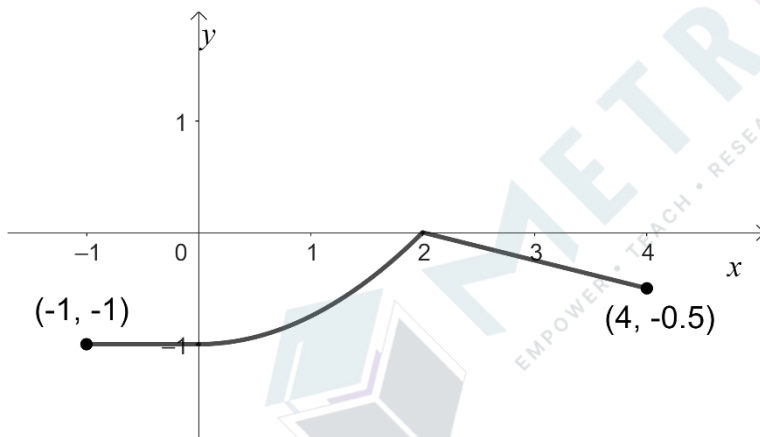
$$\begin{aligned} \int_{-3}^5 f(x) \, dx &= 3 \int_0^1 \sqrt{x} \, dx + 2 \left[\frac{1}{2}(2)(1) \right] + \frac{1}{2} \left(1 + \frac{1}{2} \right) (1) \\ &= 4.75 \end{aligned}$$

9

(a)(i) Graph of $y = f(x)$ for $-1 \leq x \leq 4$



(a)(ii) Graph of $y = f\left(\frac{1}{2}x\right) - 1$, for $-1 \leq x \leq 4$



The end-points are at $(-1, -1)$ and $(4, -0.5)$

Note: When $x = 4$

$$y = f\left(\frac{1}{2}x\right) - 1 = f(2) - 1 = -\frac{1}{2}(2) + \frac{3}{2} - 1 = -\frac{1}{2}$$

(b) Scale parallel to y -axis by factor 3.

Translate 2 units in the negative x -direction.

Reflect about the y -axis.

OR

Reflect about the y -axis.

Translate 2 units in the positive x -direction.

10

(a)
$$y = \frac{4x-9}{5-2x} = -2 + \frac{1}{5-2x}$$
$$+ 2 = \frac{1}{5-2x}$$

Replace y with $(y-2)$, we have $y = \frac{1}{5-2x}$.

Replace x with $\frac{1}{2}x$, we have $y = \frac{1}{5-x}$.

Replace x with $(x+5)$, we have $y = \frac{1}{5-(x+5)} = -\frac{1}{x}$.

- (1) Translate the graph in the positive y -direction by 2 units.
- (2) Scale the graph parallel to the x -axis by a factor 2.
- (3) Translate the graph in the negative x -direction by 5 units.

Alternatively,

Replace y with $(y-2)$, we have $y = \frac{1}{5-2x}$.

Replace x with $(x+2.5)$, we have $y = \frac{1}{5-2(x+2.5)} = \frac{1}{-2x} \Rightarrow 2y = -\frac{1}{x}$.

Replace y with $\frac{1}{2}y$, we have $y = -\frac{1}{x}$.

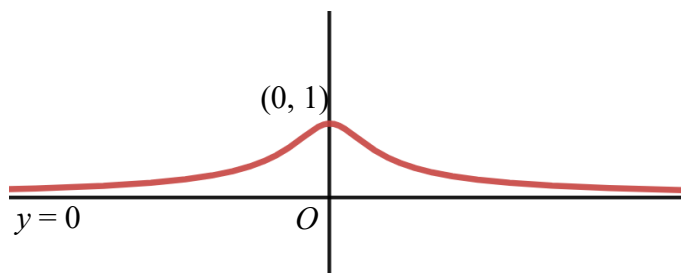
- (4) Translate the graph in the positive y -direction by 2 units.
- (5) Translate the graph in the negative x -direction by 2.5 units.
- (6) Scale the graph parallel to the y -axis by a factor 2.

(b)
$$y = \sqrt{1 + \frac{x^2}{q^2}}$$
$$y^2 = 1 + \frac{x^2}{q^2}$$

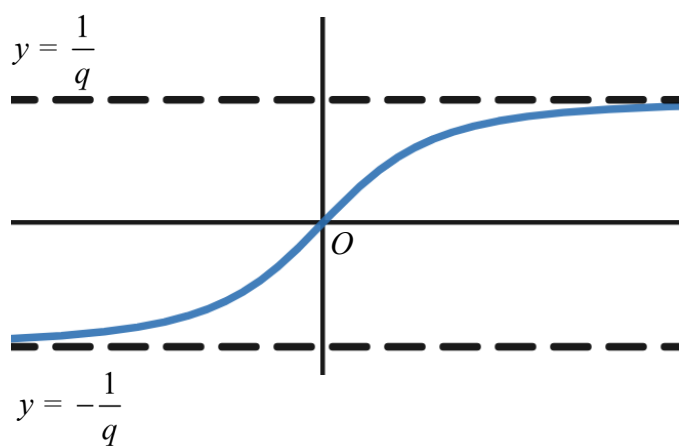
$$\frac{y^2}{1^2} - \frac{x^2}{q^2} = 1$$

It is the upper half of a hyperbola (or the part of a hyperbola above the x -axis)

(b)(i)



(b)(ii)



The original hyperbola has
asymptotes $y = \pm \frac{x}{q}$.



(a) Method 1

Reflect the graph of $y = f(x)$ about the x -axis, followed by scaling of the resulting graph by a factor of 2 parallel to the y -axis, and translating the resulting graph by 1 unit in the positive y -direction.

Method 2

Translate the graph of $y = f(x)$ by $\frac{1}{2}$ unit in the negative y -direction, followed by reflecting the resulting graph about the x -axis, and scaling by a factor of 2 parallel to the y -axis.

Method 3: Reflect the graph of $y = f(x)$ about the x -axis, followed by translating the resulting graph by $\frac{1}{2}$ unit in the positive y -direction, and scaling by a factor of 2 parallel to the y -axis.

Method 4: Scale the graph of $y = f(x)$ by a factor of 2 parallel to the y -axis, followed by translating the resulting graph by 1 unit in the negative y -direction and then reflecting about the x -axis.



(b)

A point is R -invariant if $(a, b) = \left(a, \frac{1}{b}\right) \Leftrightarrow b^2 = 1 \Leftrightarrow b = \pm 1$.

Hence if there are no R -invariant points, the graph must not intersect the lines $y = \pm 1$.

Since $f'(x) < 0$, the graph is strictly decreasing for all real values of x and some possible graphs are thus

