

1

(a) No. of ways = $4! \times 8! = 967680$

(b)(i) Method 1: Slotting technique

No. of ways = ${}^{10}C_4 \times 4! \times {}^5C_2 \times 2! \times 6! \times 2 = 145152000$

Select and arrange 4 passengers to be seated with the 2 particular girls.

No. of ways to slot in the 2 particular girls.

No. of ways to arrange the remaining 6 passengers.

Each row

Method 2

No. of ways = $({}^6C_2 - 5) \times 2! \times 10! \times 2 = 145152000$

No. of ways to arrange 2 particular girls in a row and excluding 5 possible ways (together).

Permutation between the 2 particular girls.

Arranging 10 other passengers

Each row

(b)(ii) Case 1: The 4 girls and the remaining girl are seated on the same side.

No. of ways = ${}^5C_4 \times 4! \times {}^7C_1 \times 2! \times 6! \times 2 = 2419200$

No. of ways to choose 4 girls to be together and arrange them

Choose the passenger to sit in between the 4 girls and 1 girl

Swap the position of 4 girls and 1 girl

Arrange the 6 passengers on the other side

Swap top and bottom position

Case 2: The 4 girls and the remaining girl are seated on opposite sides.

No. of ways = ${}^5C_4 \times 4! \times {}^7C_2 \times 3! \times 6! \times 2 = 21772800$

No. of ways to choose 4 girls to be together and arrange them

Choose two passengers to be at the same side as the 4

Swap the position 2 passengers and 4 girls

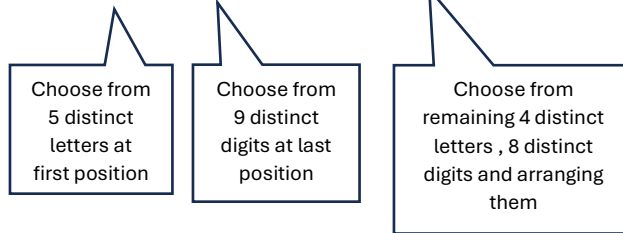
Arrange the 6 passengers on the other side

Swap top and bottom position

Total no. of ways = $2419200 + 21772800 = 24192000$

2

(a)(i) Number of ways = ${}^5C_1 \times {}^9C_1 \times {}^{12}C_5 \times 5! = 4276800$



(a)(ii) Case (1) : 3 letters + 4 digits
 ${}^5C_3 \times 3! \times {}^9C_4 \times 4! = 181440$

Case (2): 4 letters + 3 digits
 ${}^5C_4 \times 4! \times {}^9C_3 \times 3! = 60480$

Total Number of ways
 $= 181440 + 60480$
 $= 241920$

(b) All possible ways with 3 identical letters

$= {}^2C_1 \times {}^{13}C_4 \times \frac{7!}{3!} = 1201200$

“e” or “s” Choose from remaining 4 distinct letters & 9 digits

All letters and no digit

Number of ways

$= {}^2C_1 \times {}^4C_4 \times \frac{7!}{3!} = 1680$

“e” or “s” All other 4 distinct letters chosen

Since alphanumeric requires at least 1 digit,

$\therefore$ Using complement, number of ways with 3 identical letters and at least 1 digit

$= 1201200 - 1680$

$= 1199520$

3

(a) No. of results = ${}^{32}C_5 \times 5! = 24165120$

(b) No. of results

$$= \left(\begin{matrix} \text{no. of results} \\ \text{without restrictions} \end{matrix} \right) - \left(\begin{matrix} \text{no. of results} \\ \text{with traits from} \\ \text{only 1 domain} \end{matrix} \right)$$

$$= 24165120 - {}^4C_1 \times {}^8C_5 \times 5! = 24138240$$

Choose 1 of the 4 domains

Choose 5 traits from that 1 domain

(c) Case 1: Domain repeated is Cognition

$$\begin{array}{ccccccc} \text{choose 2} & \text{choose 1 trait} & \text{pick two slots} & & \text{no of ways} & & \\ \text{Cognition} & \text{each from the} & \text{for the Cognition} & & \text{to permute the} & & \\ \text{traits} & \text{non-Cognition} & \text{traits among the} & & \text{remaining 3 traits} & & \\ & \text{domain} & \text{last 3 and place} & & & & \\ & & \text{them there} & & & & \\ \text{No. of results} = & \overbrace{{}^8C_2} & \times & \overbrace{({}^8C_1)^3} & \times & \overbrace{{}^3C_2 \times 2!} & \times & \overbrace{3!} = 516096 \end{array}$$

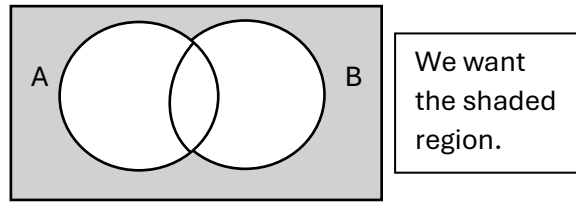
Case 2: Domain repeated is not Cognition

$$\begin{array}{ccccccc} \text{3 possible ways} & \text{Pick 2 traits} & \text{Pick 1 trait} & \text{Place} & \text{No of ways} & & \\ \text{to pick "repeating"} & \text{from "repeating"} & \text{from each of} & \text{Cognition} & \text{to permute} & & \\ \text{domain} & \text{domain} & \text{the remaining} & \text{trait in last} & \text{remaining 4} & & \\ & & \text{domains} & \text{three slots} & \text{traits} & & \\ \text{No of results} = & \overbrace{{}^3C_1} & \times & \overbrace{{}^8C_2} & \times & \overbrace{({}^8C_1)^3} & \times & \overbrace{{}^3C_1} & \times & \overbrace{4!} = 3096576 \end{array}$$

$$\text{Total no. of results} = 516096 + 3096576 = 3612672$$



(d)



A: event that Rapport traits are consecutive

B: event that Planning traits are consecutive

Method 1:

$$n((A \cup B)') = n(E) - n(A \cup B) = n(E) - (n(A) + n(B \cap A'))$$

Case 1: R consecutive

$$\text{No. of results} = 4! \times 2! = 48$$

Case 2: P consecutive, R not consecutive

$$\text{No. of results} = 2! \times {}^3C_2 \times 2! \times 2! = 24$$

Total no. of results

$$\begin{aligned} &= \left(\begin{array}{c} \text{no. of results} \\ \text{without restrictions} \end{array} \right) - \left(\begin{array}{c} \text{no. of results with} \\ \text{R consecutive} \end{array} \right) - \left(\begin{array}{c} \text{no. of results with} \\ \text{P consecutive, R not} \end{array} \right) \\ &= 5! - 48 - 24 \\ &= 48 \end{aligned}$$

Method 2:

$$n((A \cup B)') = n(E) - n(A \cup B) = n(E) - (n(A \cap B') + n(B \cap A') + n(A \cap B))$$

Complement cases are P consecutive but R not, R consecutive but P not and (R consecutive and P consecutive).

Case 1: P consecutive, R not consecutive

$$\text{No. of results} = 2! \times {}^3C_2 \times 2! \times 2! = 24$$

Case 2: R consecutive, P not consecutive

$$\text{No. of results} = \text{same as case 1} = 24$$

Case 3: R consecutive and P consecutive

$$\text{No. of results} = 3! \times 2! \times 2! = 24$$

Total no. of results

$$= 5! - 24 - 24 - 24$$

$$= 48$$

Method 3:

$$n((A \cup B)') = n(E) - n(A \cup B) = n(E) - (n(A) + n(B) - n(A \cap B))$$

Total no. of results

$$\begin{aligned} &= (\text{no. of results without restrictions}) - (\text{no. of results with R consecutive}) \\ &\quad - (\text{no. of results with P consecutive}) + (\text{no. of results with both consecutive}) \\ &= 5! - 48 - 48 + 24 \\ &= 48 \end{aligned}$$

4

Group the two first prize awardees as one object, and the two second prize awardees as another object.

1st Prizes 2nd Prizes GOH S S S S

There are 7 units to be arranged in a row.

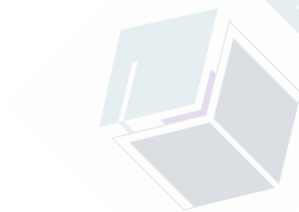
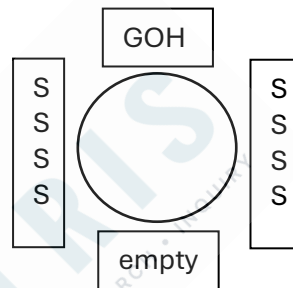
Within the unit of 1st Prize and the unit of 2nd Prize, the respective prize winners may arrange themselves.

Number of ways required = $7!2!2! = 20160$

For this arrangement, it means the guest of honour's seat opposite is an empty seat. Thus the 8 students sit at the 8 remaining chairs.

Positions are fixed, number of arrangements is the number of ways the 8 students can swap among themselves

Hence, number of seating arrangements = $8! = 40320$



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5

(i) Number of ways = ${}^4C_2 = 6$

(ii) Case 1: All same colour
 ${}^3C_1 = 3$ (excludes green bricks)

Case 2: Two different colours

${}^4C_2 \times {}^2C_1$
to choose 2 colours to choose which of the two colours picked that will be repeated

Number of ways = $6 \times 2 = 12$

Alternatively:

${}^4C_1 \times {}^3C_1 = 12$
Choose 1 of the 4 colours that repeats Choose 1 of the remaining 3 colours

Case 3: All different colours

$${}^4C_3 = 4$$

Total number of ways = $3 + 12 + 4 = 19$

(iii) Case 1: 2R 1B 1G
 ${}^5C_2 \times {}^4C_1 \times {}^2C_1 = 80$

Case 2: 2R 2Y
 ${}^5C_2 \times {}^4C_2 = 60$

Case 3: 1R 2B 1Y
 ${}^5C_1 \times {}^4C_2 \times {}^4C_1 = 120$

Case 4: 4B
 ${}^4C_4 = 1$

Total number of ways = $80 + 1 + 120 + 60 = 261$

6

- (a) Arrange the 5 boys in $(5-1)! = 24$ ways.
Then slot in each of the 3 girls into the 5 spaces between the boys in ${}^5C_3 \times 3! = 60$ ways.
Total number of arrangements with no 2 girls being adjacent to each other is $24 \times 60 = 1440$
- (b) Arrange the 3 girls within a unit in $3! = 6$ ways.
Then arrange the unit of 3 girls with the 5 boys in $(6-1)! = 120$ ways.
Total number of arrangements with all 3 girls seated together is $6 \times 120 = 720$
- (c) Total number of arrangements with exactly 2 of the 3 girls adjacent to each other is
 $= (\text{no. of ways without restrictions}) - (\text{no. of ways no two girls are beside each other})$
 $\quad - (\text{no. of ways all three girls are together})$
 $= (8-1)! - 1440 - 720$
 $= 2880$



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7

- (i) Required number of ways to have WCW, with M all separated
 = consider all W separated
 = $(4-1)! \times 4!$
 = 144

Alternatively

Case 1: 2 groups of WCW.

Number of ways to form first group of WCW = ${}^4C_2 \times {}^2C_1 \times 2! = 24$

Number of ways to form second group of WCW = ${}^2C_2 \times {}^1C_1 \times 2! = 2$

Positions of all 4 units present are fixed with respect to the circle.

Number of ways to slot the 2 men in their positions = $2!$

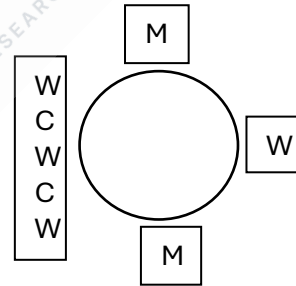
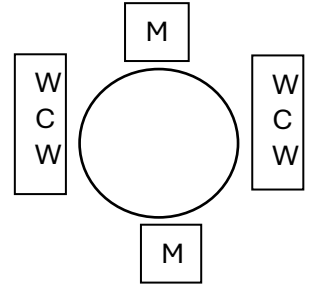
Number of ways

$$= \frac{24 \times 2 \times (2!)}{(2!)} = 48 \quad (\text{divide by } 2! \text{ as there are 2 groups of WCW})$$

Case 2: 1 group of WCWCW

Number of ways = ${}^4C_3 (3!)(2!) \times (2-1)! ({}^2C_2 \times 2!) = 96$

Total number of ways = $48 + 96 = 144$



- (ii) Number of ways = ${}^8C_4 \times (4!) = 1680$ (choose and arrange the positions of the non-women)

Alternatively,

Number of ways = $\frac{8!}{4!} = 1680$ (since $4!$ ways to arrange women if there is no restriction on them.)

- (iii) Number of ways = $\frac{{}^2C_1 \times {}^4C_2 \times {}^2C_1}{2!} = 12$ (divide by $2!$ as there are 2 groups of CWWM)