

1

(i) $X \sim B(28, 0.96)$

$$P(X = 28) = 0.31886 = 0.319 \text{ (3 s.f.)}$$

(ii)

x	$P(X = x)$
26	0.20925
27	0.37200
28	0.31886

The most likely number of students who are present in school on a typical school day is 27.

(iii) If there are more than 2 students not present, it means that there are less than 26 students present.

$$\begin{aligned}
 P(X < 26) &= P(X \leq 25) \\
 &= 0.099896 \text{ (5 s.f.)} \\
 &= 0.0999 \text{ (3 s.f.)}
 \end{aligned}$$

(iv) Let W be the number of classes where there was an investigation, out of 16. Then $W \sim B(16, 0.099896)$.

$$\begin{aligned}
 P\left(W \geq \frac{16}{3}\right) &= P(W \geq 6) \\
 &= 1 - P(W \leq 5) \\
 &= 0.0032794 \text{ (5 s.f.)} \\
 &= 0.00328 \text{ (3 s.f.)}
 \end{aligned}$$

2

- (i) Let Y be random variable “number of yellow chips in a box”.

$$Y \sim B(36, 0.3)$$

$$P(Y \leq 9) = 0.32544$$

$$P(Y > 4 | Y \leq 9) = \frac{P(5 \leq Y \leq 9)}{P(Y \leq 9)} = \frac{P(Y \leq 9) - P(Y \leq 4)}{P(Y \leq 9)} = 0.978152 \approx 0.978 \text{ (3 sf.)}$$

- (ii) Let W be random variable “number of boxes with at most 9 yellow chips”.

$$W \sim B(59, 0.32544)$$

$$\text{Required probability} = P(W = 13) \times 0.32544 = 0.00825$$

- (iii) Let X be random variable “number of boxes containing at most 9 yellow chips in a carton”.

$$X \sim B(75, 0.32544)$$

$$E(X) = 75 \times 0.32544 = 24.408$$

$$\text{Var}(X) = 75 \times 0.32544 \times (1 - 0.32544) = 16.46466$$

Since $n = 30$ is large, $\bar{X} \sim N(24.408, \frac{16.46466}{30})$ approximately by Central Limit Theorem.

$$P(\bar{X} > 25) = 0.21211 \approx 0.212 \text{ (3 sf.)}$$



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3

- (a) Let X be the number of defective phones out of 10 E-phones

$$X \sim B(10, 0.01)$$

$$P(1 \leq X < 3) = P(X \leq 2) - P(X = 0) \quad \text{or } P(X = 1) + P(X = 2) \\ = 0.0955$$

- (b) Let Y be the number of non-defective E-phones out of 9 E-phones

$$Y \sim B(9, 0.99)$$

$$P(10^{\text{th}} \text{ E-phone is the } 8^{\text{th}} \text{ phone non-defective})$$

$$= P(1^{\text{st}} 9 \text{ E-phones consist of } 7 \text{ non-defective ones}) \times P(\text{last one being non-defective})$$

$$= P(Y = 7) \times (0.99)$$

$$= (0.0033554)(0.99)$$

$$= 0.00332188$$

$$= 0.00332$$

- (c) Let W be the number of defective E-phones out of 24 E-phones

$$W \sim B(24, 0.01)$$

$$P(\text{carton rejected}) = P(W \geq 2) \\ = 1 - P(W \leq 1) \\ = 0.0238544 = 0.0239$$

(d) Let G be the number of defective phones out of 20 Galaxy phones

$$G \sim B(20, 0.015)$$

$$E(G) = 20 \times 0.015 = 0.3$$

$$\text{Var}(G) = 20 \times 0.015 \times 0.985 = 0.2955$$

$$E(W) = 24 \times 0.01 = 0.24$$

$$\text{Var}(W) = 24 \times 0.01 \times 0.99 = 0.2376$$

Let $S = G_1 + G_2 + \dots + G_{500}$ and $T = W_1 + W_2 + \dots + W_{250}$

$$E(S) = 500 \times 0.3 = 150$$

$$\text{Var}(S) = 500 \times 0.2955 = 147.75$$

$$E(T) = 250 \times 0.24 = 60$$

$$\text{Var}(T) = 250 \times 0.2376 = 59.4$$

Since 250 and 500 are large, by Central Limit Theorem,

$S \sim N(150, 147.75)$ and $T \sim N(60, 59.4)$ approximately.

Therefore, $S - T \sim N(90, 207.15)$ approximately

$P(\text{Number of defective Galaxy phones exceed that of E-phones by at least 100})$

$$= P(S - T \geq 100)$$

$$= 0.244$$

Alternative method

Let S be the no of defective phones out of 10000 Galaxy phones (500 cartons of 20 phones)

$$S \sim B(10000, 0.015)$$

$$E(S) = 10000 \times 0.015 = 150$$

$$\text{Var}(S) = 10000 \times 0.015 \times 0.985 = 147.75$$

Let T be the no of E-phones out of 6000 E-phones (250 cartons of 24 phones)

$$T \sim B(6000, 0.01)$$

$$E(T) = 6000 \times 0.01 = 60$$

$$\text{Var}(T) = 6000 \times 0.01 \times 0.99 = 59.4$$

Since 6000 and 10000 are large, by Central Limit Theorem,

$S \sim N(150, 147.75)$ and $T \sim N(60, 59.4)$ approximately.

Therefore, $S - T \sim N(90, 207.15)$ approximately

$P(\text{Number of defective Galaxy phones exceed that of E-phones by at least 100})$

$$= P(S - T \geq 100)$$

$$= 0.244$$

4

- (a) The probability of an orange being rotten is constant at $p\%$.
The event of an orange being rotten is independent of the event of any other orange being rotten.

- (b)(i) Let X be the random variable denoting the number of rotten oranges out of 10 oranges.
(defined by the question)

$$X \sim B(10, 0.2)$$

$$\begin{aligned} P(X \geq 2) &= 1 - P(X \leq 1) \\ &= 0.6241903616 \\ &= 0.624 \text{ (to 3 s.f.)} \end{aligned}$$

- (b)(ii) Expected number of packets of oranges that contains more than 1 rotten orange
 $= 100[P(X > 1)]$
 $= 100[1 - P(X \leq 1)]$
 $= 62.41903616$

$$\text{Expected profit when all the packets of oranges are sold} = 2(100) = 200$$

For the store manager to have a net profit,

$$\begin{aligned} \text{Expected loss} &< \text{Expected profit} \\ 62.41903616d &< 200 \\ d &< 3.204150726 \\ d &< 3.20 \text{ (to 2 d.p.)} \end{aligned}$$

$$\therefore 0 < d < 3.20$$

- (c) Let X be the random variable denoting the number of rotten oranges out of 10 oranges.
(defined by the question)

$$X \sim B(10, 0.01p)$$

$$P(X \leq 1) = 0.95$$

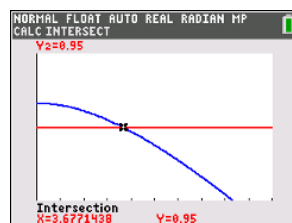
$$P(X = 0) + P(X = 1) = 0.95$$

$${}^{10}C_0 (0.01p)^0 (1 - 0.01p)^{10} + {}^{10}C_1 (0.01p)^1 (1 - 0.01p)^9 = 0.95$$

$$(1 - 0.01p)^{10} + 0.1p(1 - 0.01p)^9 = 0.95$$

Using G.C.,

$$\begin{aligned} p &= 3.6771438 \\ &= 3.68 \text{ (to 3 s.f.)} \end{aligned}$$



5

(a) Whether or not a vase is defective is independent of other vases being defective.
The probability that a vase is defective is a constant.

(b) $X \sim B(30, 0.04)$

$$\begin{aligned} P(X > 2) &= 1 - P(X \leq 2) \\ &= 0.11690 = 0.117 \text{ (3 sf)} \end{aligned}$$

(c) Let Y be the no. of days where more than 2 defective vases are found, out of 5 days.

$$Y \sim B(5, 0.11690)$$

$$P(Y \leq 1) = 0.893$$

(d) Let V be the total no. of defective vases found in 5 days.

$$V \sim B(150, 0.04)$$

Required probability

$$\begin{aligned} &P(X_1 = 4)P(X_2 = 1)\left[P(X_3 = 0)\right]^3 \times \frac{4!}{3!} \times 2! \\ &+ P(X_1 = 3)P(X_2 = 2)\left[P(X_3 = 0)\right]^3 \times \frac{4!}{3!} \times 2! \\ &= \frac{\quad}{P(V = 5)} \\ &= \frac{0.0056980}{0.16280} = 0.0350 \end{aligned}$$



6

(a) Whether or not Charlie answers a question correctly is independent of other questions.

(b) $C \sim B(30, 0.25)$

$$P(C \leq 10) = 0.894$$

(c) Let M be the number of marks Charlie can achieve

$$M = 3C - (30 - C) = 4C - 30$$

$$E(M) = E(4C - 30)$$

$$= 4E(C) - 30$$

$$= 4(30 \times 0.25) - 30$$

$$= 0 \text{ (shown)}$$

$$\text{Var}(M) = \text{Var}(4C - 30)$$

$$= 16\text{Var}(C)$$

$$= 16(30 \times 0.25 \times 0.75)$$

$$= 90$$

(d) $P(M \geq 32)$

$$= P(4C - 30 \geq 32)$$

$$= P(C \geq 15.5)$$

$$= 1 - P(C \leq 15)$$

$$= 0.000819$$



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7

- (a) $X = 8$ means there are 8 steps forward and 2 steps backward. Hence the counter ends up 6 steps ahead of point S.

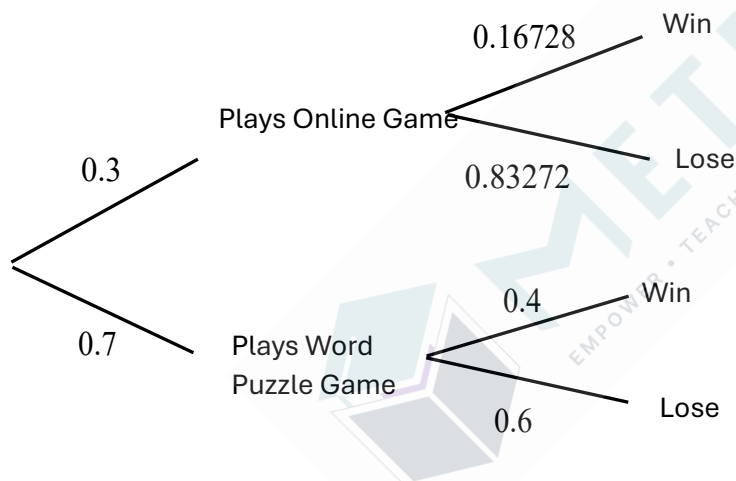
$$X \sim B\left(10, \frac{3}{5}\right)$$

$$P(X = 8) = 0.121 \quad (3 \text{ s.f.})$$

- (b) $P(\text{Serene wins the game}) = P(X \geq 8)$
 $= 1 - P(X \leq 7)$
 $= 0.16728$
 $\approx 0.167 \quad (3 \text{ s.f.}) \quad (\text{shown})$

- (c) Let W be the number of games, out of 20, that Serene wins. $W \sim B(20, 0.16728)$
 $P(W = 3) = 0.23753 \approx 0.238 \quad (3 \text{ s.f.})$

(d)



$$P(\text{Plays Word Puzzle} | \text{Wins})$$

$$= \frac{(0.7)(0.4)}{(0.7)(0.4) + (0.3)(0.16728)}$$

$$= 0.84801$$

$$\approx 0.848 \quad (3 \text{ s.f.})$$

8

(a) $X \sim B(30, p)$

$$P(X = 6) > P(X = 5)$$

$$\binom{30}{6} p^6 (1-p)^{24} > \binom{30}{5} p^5 (1-p)^{25}$$

$$25p > 6(1-p)$$

$$p > \frac{6}{31}$$

$$\therefore \frac{6}{31} < p < \frac{7}{31}$$

$$P(X = 6) > P(X = 7)$$

$$\binom{30}{6} p^6 (1-p)^{24} > \binom{30}{7} p^7 (1-p)^{23}$$

$$7(1-p) > 24p$$

$$p < \frac{7}{31}$$

and

(b) $X \sim B(30, 0.2)$

$$P(5 < X \leq 10) = P(X \leq 10) - P(X \leq 5)$$

$$= 0.54687$$

$$= 0.547 \text{ (to 3 s.f.)}$$

(c) $\bar{X} = \frac{X_1 + X_2 + \dots + X_{40}}{40}$

$$E(X) = 30(0.2) = 6$$

$$\text{Var}(X) = 30(0.2)(0.8) = 4.8$$

Since $n = 40$ is large, by Central Limit Theorem, $\bar{X} \sim N\left(6, \frac{4.8}{40}\right)$ approximately.

$$P(\bar{X} \geq 5) = 0.99805$$

$$= 0.998 \text{ (to 3 s.f.)}$$



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9

(a)
$$P(\text{Obtaining 'hit'}) = \frac{\text{Area of inner circle}}{\text{Area of outer circle}}$$

$$= \frac{\pi r^2}{\pi(15^2)}$$

$$= \frac{r^2}{225}$$

(b)(i) Let X be the random variable denoting the number of 'hits' out of 8 throws. Then

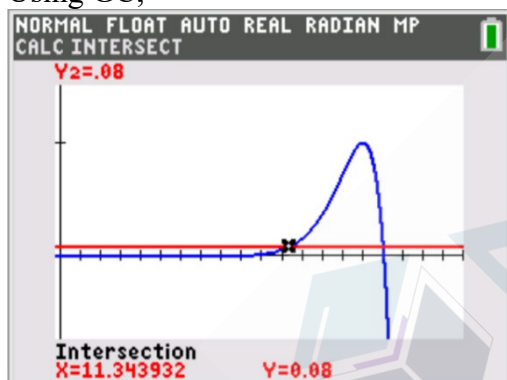
$$X \sim B\left(8, \frac{r^2}{225}\right)$$

$$P(X > 6) \leq 0.08$$

$$P(X = 7) + P(X = 8) \leq 0.08$$

$${}^8C_7 \left(\frac{r^2}{225}\right)^7 \left(1 - \frac{r^2}{225}\right) + \left(\frac{r^2}{225}\right)^8 \leq 0.08$$

Using GC,



$$r \leq 11.3439 \text{ (6 s.f.)}$$

Therefore, $r \leq 11.3$ (3 s.f.)

(b)(ii)
$$\frac{6^2}{225} = 0.16$$

Let Y be the random variable denoting the number of 'hits' out of 4 throws. Then

$$Y \sim B(4, 0.16)$$

Required probability

$$= P(Y = 1) \times (0.16)$$

$$= 0.060693$$

$$= 0.0607 \text{ (3 s.f.)}$$

Alternative solution

Required probability

$$= [(0.16)(0.84)^3 \times 4](0.16)$$

$$= 0.0607 \text{ (3 s.f.)}$$

(b)(iii) Let W be the random variable denoting the number of 'hits' out of n throws. Then $W \sim B(n, 0.16)$

$$P(W \geq 1) \geq 0.7$$

$$1 - P(W = 0) \geq 0.7$$

$$P(W = 0) \leq 0.3$$

By GC,

n	$P(W = 0)$
6	$0.3513 > 0.3$
7	$0.2951 < 0.3$
8	$0.2479 < 0.3$

Therefore, the least value of n is 7 .

(b)(iv) $X \sim B(8, 0.16)$

Required probability

$$= P(X < 2(8 \times 0.16) | X > 8 \times 0.16)$$

$$= P(X < 2.56 | X > 1.28)$$

$$= \frac{P(1.28 < X < 2.56)}{P(X > 1.28)}$$

$$= \frac{P(X = 2)}{P(X \geq 2)}$$

$$= \frac{0.25181}{1 - 0.62559}$$

$$= 0.673 \text{ (3 s.f.)}$$



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10

- (i) The probability that a screen protector is cracked is constant at 0.04.
The event that a screen protector is cracked is independent of any other screen protector.

- (ii) Let X = number of cracked screen protectors out of n
 $X \sim B(n, 0.04)$

Given that mode is 2,

$$P(X=1) < P(X=2)$$

and

$$P(X=3) < P(X=2)$$

$$\begin{aligned} \binom{n}{1}(0.04)^1(0.96)^{n-1} &< \binom{n}{2}(0.04)^2(0.96)^{n-2} \\ \frac{n!}{1!(n-1)!}(0.04)(0.96)^{n-1} &< \frac{n!}{2!(n-2)!}(0.04)^2(0.96)^{n-2} \\ \frac{2!(0.96)^{n-1}}{1!(0.96)^{n-2}} &< \frac{(n-1)!(0.04)^2}{(n-2)! \cdot 0.04} \\ 2(0.96) &< (n-1)(0.04) \\ n &> 49 \\ \binom{n}{3}(0.04)^3(0.96)^{n-3} &< \binom{n}{2}(0.04)^2(0.96)^{n-2} \\ \frac{n!}{3!(n-3)!}(0.04)^3(0.96)^{n-3} &< \frac{n!}{2!(n-2)!}(0.04)^2(0.96)^{n-2} \\ \frac{(n-2)!(0.04)^3}{(n-3)!(0.04)^2} &< \frac{3!(0.96)^{n-2}}{2!(0.96)^{n-3}} \\ (n-2)(0.04) &< 3(0.96) \\ n &< 74 \\ \therefore 49 < n < 74 \quad n \in \mathbb{Z} \end{aligned}$$

- (iii) Y = number of cracked screen protectors out of 50
 $Y \sim B(50, 0.04)$

Required Probability = $P(Y \leq 3)$

$$= 0.86087 \approx 0.861 \quad (3 \text{ s.f.})$$

- (iv) Required Probability
 $= P(Y > 2 \mid Y \leq 6)$

$$= \frac{P(Y > 2 \cap Y \leq 6)}{P(Y \leq 6)}$$

$$= \frac{P(2 < Y \leq 6)}{P(Y \leq 6)}$$

$$= \frac{P(Y \leq 6) - P(Y \leq 2)}{P(Y \leq 6)}$$

$$= \frac{0.99639 - 0.67671}{0.99639}$$

$$\approx 0.321 \quad (3 \text{ s.f.})$$

- (v) Required Probability

$$= {}^6C_3 \times (0.86087)^3 \times (1 - 0.86087)^3 \times (0.86087)$$

$$\approx 0.0296 \quad (3 \text{ s.f.})$$

11

Let X be the number of students who study H2 Biology out of 40 students.

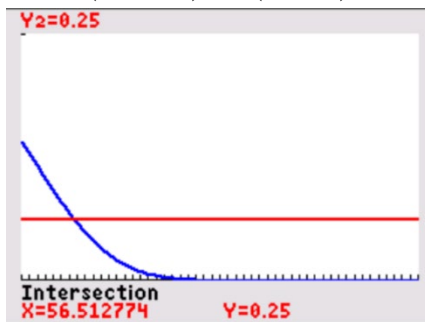
$$X \sim B\left(40, \frac{p}{100}\right)$$

$$P(9 \leq X \leq 20) = 0.25$$

$$P(X \leq 20) - P(X \leq 8) = 0.25$$

Obtain graphs of these below using GC:

$$y = P(X \leq 20) - P(X \leq 8) \text{ and } y = 0.25$$



$$p = 56.5 \text{ (3 s.f.)}$$

Let A be the number of students out of 6 students out of the 4th to 9th interviewees who study H2 Biology.

$$A \sim B(6, 0.6)$$

Required probability

$$\begin{aligned} &= (0.4)^2 (0.6) P(A=2) (0.6) \\ &= 0.00796 \text{ (3 s.f.)} \end{aligned}$$

12

- (i) The probability of a student achieving distinction in the course is a constant
OR

The event that a student achieves distinction is independent of the event that another student achieves distinction.

(ii) $X \sim B\left(10, \frac{p}{100}\right)$

Mode of X is 7.

$$P(X = 6) < P(X = 7)$$

$$\binom{10}{6} \left(\frac{p}{100}\right)^6 \left(1 - \frac{p}{100}\right)^4 < \binom{10}{7} \left(\frac{p}{100}\right)^7 \left(1 - \frac{p}{100}\right)^3$$

$$\frac{10!}{6!4!} \left(\frac{p}{100}\right)^6 \left(1 - \frac{p}{100}\right)^4 < \frac{10!}{7!3!} \left(\frac{p}{100}\right)^7 \left(1 - \frac{p}{100}\right)^3$$

$$7 \left(1 - \frac{p}{100}\right) < 4 \left(\frac{p}{100}\right)$$

$$p > \frac{700}{11}$$

and

$$P(X = 7) > P(X = 8)$$

$$\frac{10!}{7!3!} \left(\frac{p}{100}\right)^7 \left(1 - \frac{p}{100}\right)^3 > \frac{10!}{8!2!} \left(\frac{p}{100}\right)^8 \left(1 - \frac{p}{100}\right)^2$$

$$8 \left(1 - \frac{p}{100}\right) > 3 \left(\frac{p}{100}\right)$$

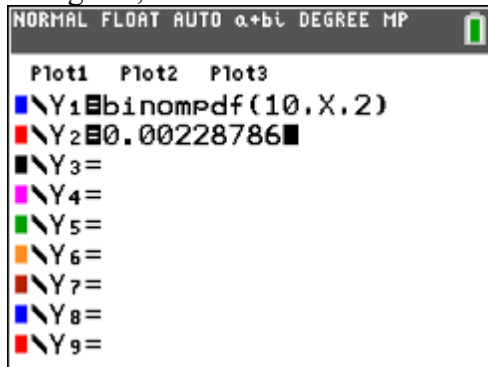
$$p < \frac{800}{11}$$

$$\Rightarrow \frac{700}{11} < p < \frac{800}{11}$$

$$\Rightarrow 63.6 < p < 72.7 \text{ (3sf)}$$

(iii) $P(X = 2) = 0.00228786$

Using GC,



Note that since $63.6 < p < 72.7$, we should look for probabilities above 0.5.



$$\therefore \frac{p}{100} = 0.68$$

$$\Rightarrow p = 68$$

(iv) $X \sim B(10, 0.68)$

$$P(X > 6) = 1 - P(X \leq 6) = 0.595637$$

Let Y be the number of concluded courses out of 8 courses, with more than 6 students achieving distinction

$$Y \sim B(8, 0.595637)$$

$$\begin{aligned} P(Y \leq 7 | Y \geq 5) &= \frac{P(5 \leq Y \leq 7)}{P(Y \geq 5)} \\ &= \frac{P(Y \leq 7) - P(Y \leq 4)}{1 - P(Y \leq 4)} \\ &= \frac{0.984156 - 0.416069}{1 - 0.416069} \\ &\approx 0.973 \text{ (3sf)} \end{aligned}$$

13

- (i) The probability that a patient has diabetes mellitus is a constant at $\frac{1}{15}$ for each patient.
A patient having diabetes mellitus is independent of any other patient having diabetes mellitus.

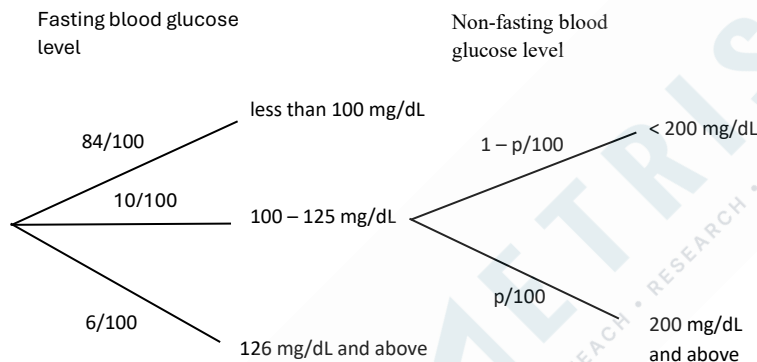
- (ii) Let Y be the random variable “number of patients having diabetes mellitus out of 49.”

$$Y \sim B\left(49, \frac{1}{15}\right)$$

Required Probability

$$\begin{aligned} &= P(Y = 3)P(\text{the 50th patient is the fourth patient who has diabetes}) \\ &= 0.2284556 \times \frac{1}{15} = 0.0152 \end{aligned}$$

- (iii)



$$\begin{aligned} \text{Required Probability} &= \frac{6}{100} + \frac{10}{100} \times \frac{p}{100} \\ &= 0.06 + 0.001p \end{aligned}$$

- (iv) $P(100 - 125 \text{ mg/dL} \mid \text{not diagnosed with diabetes mellitus})$

$$= \frac{P(100 - 125 \text{ mg/dL} \cap \text{not diagnosed with diabetes mellitus})}{P(\text{not diagnosed with diabetes mellitus})}$$

$$= \frac{\frac{10}{100} \times \left(1 - \frac{p}{100}\right)}{1 - (0.06 + 0.001p)}$$

$$= \frac{0.1 - 0.001p}{0.94 - 0.001p} = \frac{100 - p}{940 - p}$$

- (v) Let W be the random variable “number of patients having diabetes mellitus out of n .”

When $p = 20$, $0.06 + 0.001p = 0.08$

$$W \sim B(n, 0.08)$$

$$P(W \geq 6) > 0.3$$

$$1 - P(W \leq 5) > 0.3$$

$$P(W \leq 5) < 0.7$$

From GC,

n	$P(W \leq 5)$
56	$0.7102 > 0.7$
57	$0.696 < 0.7$
58	$0.6816 < 0.7$

Least value of $n = 57$



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(a) Consider 2 cases:

Case 1: 4th woman (W4) holds white or red light stick

W4: 2 choices

W3 and W5: 3 choices each

W1, W2, W6, W7: 2 choices each

No. of ways: $2 \times 3^2 \times 2^4 = 288$

Case 2: 4th woman (W4) holds pink, blue or green light stick

W4: 3 choices

W3 and W5: 2 choices each

W2 and W6: 2 choices each

W1 and W7: 2 choices each

No. of ways: $3 \times 2^2 \times 2^2 \times 2^2 = 192$

Total = $288 + 192 = 480$

(b)(i) Let X be no. of fans, out of 7, whose birthdays are in September.

$$X \sim B\left(7, \frac{1}{12}\right)$$

$$P(X = 2) = 0.094387 = 0.0944$$

(b)(ii) Let W be no. of groups of fans, out of 20, with exactly 2 fans having birthdays in September.

$$W \sim B(20, 0.094387)$$

$$P(W \geq 3) = 1 - P(W \leq 2) = 0.291$$

(b)(iii) P(the child stands between any two women)

$$= \frac{{}^4C_2 \times (5-1)! \times 2!}{(7-1)!} = \frac{2}{5} \quad (\text{or } 0.4)$$

(b)(iv) Let Y be no. of fans, out of 120, whose birthdays are in June, July, August or September.

$$Y \sim B\left(120, \frac{4}{12}\right)$$

$$0.4 \times 120 = 48$$

$$P(Y \leq 48) = 0.948$$

15

(a) $P(R_2 = 1) = P(\text{both scores are odd numbers}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$$P(R_2 = 0) = 1 - \frac{1}{4} = \frac{3}{4}$$

OR:

R_2	1	2	3	4	5	6
1	1	0	1	0	1	0
2	0	0	0	0	0	0
3	1	0	1	0	1	0
4	0	0	0	0	0	0
5	1	0	1	0	1	0
6	0	0	0	0	0	0

OR:

$D_1 \times D_2$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

$$P(R_2 = 1) = \frac{9}{36} = \frac{1}{4}, \quad P(R_2 = 0) = \frac{27}{36} = \frac{3}{4}$$

Probability distribution table for R_2 :

r_2	0	1
$P(R_2 = r_2)$	$\frac{3}{4}$	$\frac{1}{4}$

$$E(R_2) = 0 \times \frac{3}{4} + 1 \times \frac{1}{4} = \frac{1}{4}$$

(b) $R_6 = 0$ when the product is divisible by 6.

R_6	1	2	3	4	5	6
1						0
2			0			0
3		0		0		0
4			0			0
5						0
6	0	0	0	0	0	0

Alternatively,

$D_1 \times D_2$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

$$P(R_6 = 0) = P(\text{products divisible by 6}) = \frac{15}{36} = \frac{5}{12} \text{ (shown)}$$

(c) $X \sim B\left(m, \frac{5}{12}\right)$

$$\begin{aligned} P(X=2) &= \frac{m!}{2!(m-2)!} \left(\frac{5}{12}\right)^2 \left(\frac{7}{12}\right)^{m-2} \\ &= \frac{m(m-1)}{2} \left(\frac{5}{12}\right)^2 \left(\frac{7}{12}\right)^m \left(\frac{12}{7}\right)^2 \\ &= \frac{25}{98} m(m-1) \left(\frac{7}{12}\right)^m \quad \text{where } A = \frac{25}{98} \text{ (shown)} \end{aligned}$$

Given $\frac{25}{98} m(m-1) \left(\frac{7}{12}\right)^m = 0.03604$

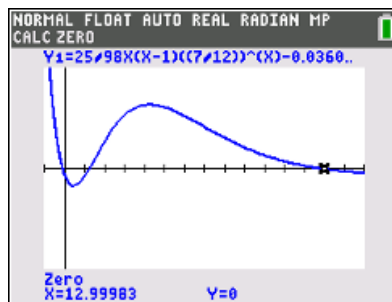
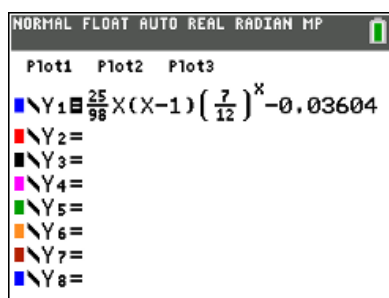
From GC, $m = -0.11850, 1.2233$ or 13.000

Since $m \geq 2$, $m = 13$ (Note: $m \in \mathbb{Z}^+$)

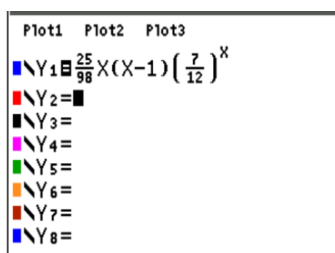


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OR Since m is an integer, we can choose to use table to find the integer m that best satisfies the equation.



NORMAL FLOAT AUTO REAL RADIAN MP				
PRESS $\blacktriangle$ TO EDIT FUNCTION				
X	Y1			
4	0.3545			
5	0.3446			
6	0.3015			
7	0.2463			
8	0.1915			
9	0.1436			
10	0.1047			
11	0.0747			
12	0.0523			
13	0.036			
14	0.0245			
Y1=0.03603768234804				

m	$P(X = 2)$
12	0.05227
13	0.03604, which is closest to 0.03604
14	0.02453

Thus $m = 13$ since m is an integer.

- (d) Let C and D denote the number of products that are divisible by 6 obtained in 10 throws of 2 fair dice by a student from Class C and a student from class D respectively.

$$C, D \sim B\left(10, \frac{5}{12}\right)$$

$$E(C) = E(D) = 10 \times \frac{5}{12} = \frac{25}{6} \text{ or } 4.1667 \text{ (5sf)}$$

$$\text{Var}(C) = \text{Var}(D) = 10 \times \frac{5}{12} \times \frac{7}{12} = \frac{175}{72} \text{ or } 2.4306 \text{ (5sf)}$$

Since sample sizes, 35 and 30 are large, by Central Limit Theorem, $\bar{C} \sim N\left(\frac{25}{6}, \frac{175}{72(35)} = \frac{5}{72}\right)$

approximately and

$$\bar{D} \sim N\left(\frac{25}{6}, \frac{175}{72(30)} = \frac{35}{432}\right) \text{ approximately.}$$

$$\therefore \bar{C} - \bar{D} \sim N\left(0, \frac{5}{72} + \frac{35}{432} = \frac{65}{432}\right) \text{ approximately.}$$

$$P(\bar{C} - \bar{D} > 0.2) = 0.303 \text{ (3sf)}$$

OR (Less preferred)

Let T_c and T_D be the total number of products divisible by 6 for class C and class D respectively.

$$T_c \sim B\left(350, \frac{5}{12}\right) \text{ and } T_D \sim B\left(300, \frac{5}{12}\right).$$

Now since sample sizes of 350 and 300 are both large, by Central Limit Theorem,

$$T_c \sim N\left(350\left(\frac{5}{12}\right), 350\left(\frac{5}{12}\right)\left(\frac{7}{12}\right)\right) \text{ approximately, and}$$

$$T_D \sim N\left(300\left(\frac{5}{12}\right), 300\left(\frac{5}{12}\right)\left(\frac{7}{12}\right)\right) \text{ approximately}$$

$$E\left(\frac{T_c}{35} - \frac{T_D}{30}\right) = 0 \text{ and } \text{Var}\left(\frac{T_c}{35} - \frac{T_D}{30}\right) = \frac{1}{35^2} \text{Var}(T_c) + \frac{1}{30^2} \text{Var}(T_D)$$

$$\frac{T_c}{35} - \frac{T_D}{30} \sim N\left(0, \frac{65}{432}\right) \text{ approximately}$$

$$P\left(\frac{T_c}{35} - \frac{T_D}{30} > 0.2\right) = 0.303 \text{ (3sf)}$$



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- (a) For a player to reach point B , there must be 5 right steps and 3 up steps in total.
Let R_1 be the number of right steps taken by a player out of 5 to move from X to Y .

$$\begin{aligned} R_1 &\sim B(5, p) \\ P(R_1 = 3) \\ &= {}^5C_3 \times p^3 q^2 \\ &= 10p^3 q^2 \end{aligned}$$

Let R_2 be the number of right steps taken by a player out of 3 to move from Y to B .

$$\begin{aligned} R_2 &\sim B(3, p) \\ P(R_2 = 2) \\ &= {}^3C_2 \times p^2 q \\ &= 3p^2 q \end{aligned}$$

$$\begin{aligned} \text{Required probability} \\ &= 10p^3 q^2 \times 3p^2 q \\ &= 30p^5 q^3 \text{ (shown)} \end{aligned}$$

(b) $W \sim B\left(15, 30\left(\frac{4}{5}\right)^5 \left(\frac{1}{5}\right)^3\right) \Rightarrow W \sim B(15, 0.078643)$

$$\begin{aligned} P(W \geq 5) \\ &= 1 - P(W \leq 4) \\ &= 0.0046167 \\ &= 0.00462 \text{ (3 s.f.)} \end{aligned}$$

(c) $W \sim B(15, 0.078643)$

$$E(W) = 15(0.078643) = 1.1796$$

$$\text{Var}(W) = 1.1796(1 - 0.078643) = 1.0869$$

Since sample size = 40 is large, by Central Limit Theorem,

$$\bar{W} = \frac{W_1 + \dots + W_{40}}{40} \sim N\left(1.1796, \frac{1.0869}{40}\right) \text{ approximately.}$$

$$P(\bar{W} \leq 1) = 0.138 \text{ (3 s.f.)}$$